Chapter 1
Mediation Analysis with Missing Data through Multiple Imputation and Bootstrap

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1.1 Introduction

Mediation models and mediation analysis are widely used in behavioral and social sciences as well as in health and medical research. Mediation models are very useful for theory development and testing as well as for identification of intervention points in applied work. Although mediation models were first developed in psychology (e.g., MacCorquodale & Meehl, 1948; Woodworth, 1928), they have been recognized and used in many disciplines where the mediation effect is also known as the indirect effect (Sociology, Alwin & Hauser, 1975) and the surrogate or intermediate endpoint effect (Epidemiology, Freedman & Schatzkin, 1992).

Figure 1.1 depicts the path diagram of a simple mediation model. In this figure, X, M, and Y represent the independent or input variable, the mediation variable (mediator), and the dependent or outcome variable, respectively. The e_M and e_Y are residuals or disturbances with variances σ^2_{eM} and σ^2_{eY}. The coefficient c' is called the direct effect, and the mediation effect or indirect effect is measured by the product term $ab = a \times b$ as an indirect path from X to Y through M. The other parameters in this model include the intercepts i_M and i_Y.

Statistical approaches to estimating and testing mediation effects with complete data have been discussed extensively in the psychological literature (e.g., Baron & Kenny, 1986; Bollen & Stine, 1990; MacKinnon et al., 2002, 2007; Shrout & Bolger, 2002). One way to test mediation effects is to test $H_0 : ab = 0$. If a large sample is available, the normal approximation method can be used, which constructs the standard error of $\widehat{ab}$ through the delta method so that $s.e.(\widehat{ab}) =$

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$\sqrt{\hat{b}^2\hat{\sigma_a^2}+2\hat{a}\hat{b}\hat{\sigma}_{ab}+\hat{a}^2\hat{\sigma_b^2}}$ with the parameter estimates $\hat{a}$ and $\hat{b}$, their estimated variances $\hat{\sigma_a^2}$ and $\hat{\sigma_b^2}$, and covariance $\hat{\sigma}_{ab}$ (e.g., Sobel, 1982, 1986). Many researchers suggested that the distribution of a mediation effect may not be normal especially when the sample size is small although with large sample sizes the distribution may still approach normality (Bollen & Stine, 1990; MacKinnon et al., 2002). Thus, bootstrap methods have been recommended to obtain the empirical distribution and confidence interval of a mediation effect (MacKinnon et al., 2004; Mallinckrodt et al., 2006; Preacher & Hayes, 2008; Shrout & Bolger, 2002; Zhang & Wang, 2008).

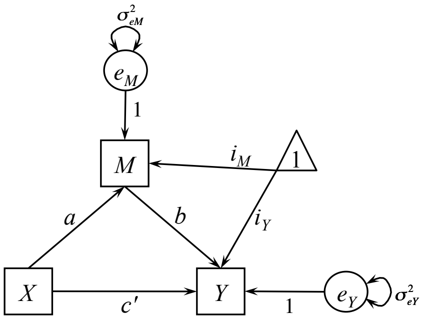


Fig. 1.1: Path diagram demonstration of a mediation model.

Missing data is continuously a challenge even for a well designed study. Although there are approaches to dealing with missing data for path analysis in general (for a recent review, see Graham, 2009), there are few studies focusing on the treatment of missing data in mediation analysis. Mediation analysis is different from typical path analysis because the focus is on the product of multiple path coefficients. A common practice is to analyze complete data through listwise deletion or pairwise deletion (e.g., Chen et al., 2005; Preacher & Hayes, 2004). Recently, Zhang & Wang (2013b) discussed how to deal with missing data in mediation analysis through multiple imputation and full information maximum likelihood. However, the number of imputations needed to get reliable results remains unclear.

In this study, we discuss how to deal with missing data for mediation analysis through multiple imputation (MI) and bootstrap. We will first present some technical aspects of multiple imputation for mediation analysis with missing data. Then, we will present two simulation studies to evaluate the performance of MI for mediation analysis with missing data. In particular, we investigate the number of imputations needed for mediation analysis. Next, an empirical example will be used to demonstrate the application of the method. Finally, we discuss the limitations of the study and future directions. Instructions on how to use SAS and R to conduct mediation analysis through multiple imputation and bootstrap are provided online as supplemental materials.

1.2 Method

1.2.1 Complete data mediation analysis

We focus our discussion on the simple mediation model to better illustrate the method although the approach works for the general mediation model. In its mathematical form, the mediation model displayed in Figure 1.1 can be expressed using two equations,

$$\begin{aligned} M &= i_M + aX + e_M \\ Y &= i_Y + bM + c'X + e_Y, \end{aligned} \tag{1.1}$$

which can be viewed as a collection of two linear regression models. To obtain the parameter estimates in the model, the maximum likelihood estimation method for structural equation modeling can be used. Specifically for the simple mediation model, the mediation effect estimate is $\widehat{ab} = \hat{a}\hat{b}$ with

$$\begin{aligned} \hat{a} &= s_{XM}/s_X^2 \\ \hat{b} &= (s_{MY}s_X^2 - s_{XM}s_{XY})/(s_X^2 s_M^2 - s_{XM}^2) \end{aligned} \tag{1.2}$$

where $s_X^2, s_M^2, s_Y^2, s_{XM}, s_{MY}, s_{XY}$ are sample variances and covariances of X, M, Y, respectively.

1.2.2 Missingness mechanisms

Little & Rubin (1987, 2002) have distinguished three types of missing data – missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR). Let $D = (X, M, Y)$ denote all data that can be potentially observed in a mediation model. D_{obs} and D_{miss} denote data that are actually observed and data that are not observed, respectively. Let R denote an indicator matrix of zeros and ones with the same dimension as D. If a datum in D is missing, the corresponding element in R is equal to 1. Otherwise, it is equal to 0. Finally, let A denote the auxiliary variables that are related to the missingness of D but not a component of the mediation model.

If the missing mechanism is MCAR, then we have

$$\Pr(R|D_{obs}, D_{miss}, \boldsymbol{\theta}) = \Pr(R|\boldsymbol{\theta}),$$

where the vector $\boldsymbol{\theta}$ represents all model parameters in the mediation model. This suggests that missing data D_{miss} are a simple random sample of D and not related to the data observed or auxiliary variables A. If the missing mechanism is MAR, then

$$\Pr(R|D_{obs}, D_{miss}, \boldsymbol{\theta}) = \Pr(R|D_{obs}, \boldsymbol{\theta}),$$

which indicates that the probability that a datum is missing is related to the observed data D_{obs} but not to the missing data D_{miss}.

Finally, if the probability that a datum is missing is related to the missing data D_{miss} or auxiliary variables A but A are not considered in the data analysis, the missing mechanism is MNAR. In particular, we want to emphasize the MNAR mechanism with auxiliary variables where

$$\Pr(R|D_{obs}, D_{miss}, \boldsymbol{\theta}) = \Pr(R|D_{obs}, A, \boldsymbol{\theta}).$$

Note that although the missingness is MNAR if only D is modeled, the overall missingness becomes to MAR if D and A are jointly modeled. Therefore, one way to deal with MNAR is to identify and include the auxiliary variables that are related to missingness.

1.2.3 Multiple imputation for mediation analysis with missing data

Most techniques dealing with missing data, including multiple imputation, in general require missing data to be either MCAR or MAR (see also, e.g., Little & Rubin, 2002; Schafer, 1997). For MNAR, the missing mechanism has to be known to correctly recover model parameters (e.g., Lu et al., 2011; Zhang & Wang, 2012). Practically, researchers have suggested including auxiliary variables to facilitate MNAR missing data analysis (Graham, 2003; Savalei & Bentler, 2009). After including appropriate auxiliary variables, we may be able to assume that data from both model variables and auxiliary variables are MAR.

Assume that a set of $p(p \geq 0)$ auxiliary variables $A_1, A_2, \ldots, A_p$ are available. These auxiliary variables may or may not be related to missingness of the mediation model variables. By augmenting the auxiliary variables with the mediation model variables, we have $D = (X, M, Y, A_1, \ldots, A_p)$, e.g., for the simple mediation model. To proceed, we assume that the missing mechanism is MAR after including the auxiliary variables.

Multiple imputation (Little & Rubin, 2002; Rubin, 1976; Schafer, 1997) is a procedure to fill each missing value with a set of plausible values. The multiple imputed data sets are then analyzed using standard procedures for complete data and the results from these analyses are combined for obtaining point estimates of model parameters and their standard errors. For mediation analysis with missing data, the following steps can be implemented for obtaining point estimates of mediation model parameters:

1. Assuming that $D = (X, M, Y, A_1, \ldots, A_p)$ are from a multivariate normal distribution, generate K sets of values for each missing value. Combine the generated values with the observed data to produce K sets of complete data (Schafer, 1997).
2. For each of the K sets of complete data, apply the formula in Equation 1.2 or use the structural equation modeling (SEM) method to obtain a point mediation effect estimate $\widehat{ab}_k = \hat{a}_k \hat{b}_k (j = 1, \ldots, K)$.
3. The point estimate for the mediation effect through multiple imputation is the average of the K complete data mediation effect estimates:

$$\widehat{ab} = \hat{a}\hat{b} = \frac{1}{K} \sum_{k=1}^{K} \hat{a}_k \hat{b}_k.$$

Parameter estimates for other model parameters can be obtained in the same way.

1.2.4 Testing mediation effects through the bootstrap method

The procedure described above is implemented to obtain point estimates of mediation effects. The bootstrap method has been used to test the significance of the mediation effects (e.g., Bollen & Stine, 1990). This method has no distribution assumption on the indirect effect. Instead, it approximates the distribution of the indirect effect using its bootstrap empirical distribution. The bootstrap method can be applied along with multiple imputation to obtain standard errors of mediation effect estimates and confidence intervals for mediation analysis with missing data. Specifically, the following procedure can be used.

1. Using the *original data set* (sample size = N) as a population, draw a bootstrap sample of N persons randomly with replacement from the original data set. This bootstrap sample generally would contain missing data.
2. With the bootstrap sample, implement the K multiple imputation procedure described in the above section to obtain point estimates of model parameters and a point estimate of the mediation effect.
3. Repeat Steps 1 and 2 for a total of B times. B is called the number of bootstrap samples.
4. Empirical distributions of model parameters and the mediation effect are then obtained using the B sets of bootstrap point estimates. Thus, confidence intervals of model parameters and the mediation effect can be constructed.

The procedure described above can be considered as a procedure of K multiple imputations nested within B bootstrap samples. Using the B bootstrap sample point estimates, one can obtain bootstrap standard errors and confidence intervals of model parameters and mediation effects conveniently. Let $\boldsymbol{\theta}$ denote a vector of model parameters and the mediation effects. With data from each bootstrap, we can obtain $\hat{\boldsymbol{\theta}}^b$, $b = 1, \ldots, B$. The standard error estimate of the pth parameter $\hat{\theta}_p$ can be calculated as

$$\widehat{s.e.(\hat{\theta}_p)} = \sqrt{\sum_{b=1}^{B}(\hat{\theta}_p^b - \bar{\hat{\theta}}_p^b)^2/(B-1)}$$

with $\bar{\hat{\theta}}_p^b = \sum_{b=1}^{B} \hat{\theta}_p^b/B$.

Many methods for constructing confidence intervals from $\hat{\boldsymbol{\theta}}^b$ have been proposed such as the percentile interval, the bias-corrected (BC) interval, and the bias-corrected and accelerated (BCa) interval (Efron, 1987; MacKinnon et al., 2004). In the present study, we focus on the BC interval because MacKinnon et al. (2004) showed that, in general, the BC confidence intervals have performed better in terms of Type I error and statistical power among many different confidence intervals. The $1-2\alpha$ BC interval for the pth element of $\boldsymbol{\theta}$ can be constructed using the percentiles $\hat{\theta}_p^b(\tilde{\alpha}_l)$ and $\hat{\theta}_p^b(\tilde{\alpha}_u)$ of $\hat{\theta}_p^b$ with $\tilde{\alpha}_l = \Phi(2z_0 + z^{(\alpha)})$ and $\tilde{\alpha}_u = \Phi(2z_0 + z^{(1-\alpha)})$ where Φ is the standard cumulative normal distribution function and $z^{(\alpha)}$ is the α percentile of the standard normal distribution and

$$z_0 = \Phi^{-1}\left[\frac{\text{number of times that } \hat{\theta}_p^b < \hat{\theta}_p}{B}\right].$$

1.3 Simulation Studies

In this section, we conduct two simulation studies to evaluate the performance of the proposed method for mediation analysis with missing data. We first evaluate its performance under different missing data mechanisms including MCAR, MAR, and MNAR without and with auxiliary variables. Then, we investigate how many imputations are needed for different proportions of missing data.

1.3.1 Simulation study 1. Estimate of mediation effects under MCAR, MAR, and MNAR data

1.3.1.1 Simulation design

For mediation analysis with complete data, simulation studies have been conducted to investigate a variety of features of mediation models (e.g., MacKinnon et al., 2002, 2004). For the current study, we follow the parameter setup from the previous literature and set the population parameter values to be $a = b = .39$, $c' = 0$, $i_M = i_Y = 0$, and $\sigma_{eM}^2 = \sigma_{eY}^2 = \sigma_{eX}^2 = 1$. Furthermore, we fix the sample size at $N = 100$ and consider three proportions of missingness with missing data percentages at 10%, 20%, and 40%, respectively. To facilitate the comparisons among different missing mechanisms, missing data are only allowed in M and Y although our software programs also allow missingness in X. Two auxiliary variables (A_1 and

A_2) are also generated where the correlation between A_1 and M and the correlation between A_2 and Y are both 0.5.

Missing data are generated in the following way. First, 1000 sets of complete data are generated. Second, for MCAR, each data value has the same probability of missing for M and Y. Third, for the MAR data, the probability of missingness in Y and M depends only on X. Specifically, if X is smaller than a given percentile, M is missing and if X is larger than a given percentile, Y is missing. Finally, to generate MNAR data, we assume that missingness of M depends on A_1 and missingness of Y depends on A_2. If A_1 is smaller than a given percentile, M is missing, and if A_2 is smaller than a percentile, Y is missing. Clearly, if auxiliary variables A_1 and A_2 are included in an analysis, the missing mechanism becomes MAR. However, if the auxiliary are not considered, the missing mechanism is MNAR.

The generated data are analyzed using the R package bmem. To estimate the mediation effects, the sample covariance matrix of the imputed data is first estimated. Then, the mediation model is fitted to the estimated covariance matrix to obtain the model parameters through the SEM maximum likelihood estimation method (Bollen, 1989). Finally, the mediation effects are calculated as the product the corresponding direct effects.

1.3.1.2 Results

The parameter estimate bias, coverage probability, and power for MCAR, MAR, and MNAR data with 10%, 20%, and 40% missing data were obtained without and with auxiliary variables and are summarized in Table 1.1. Based on the results, we can conclude the following. First, the bias of the parameter estimates under the studied MCAR conditions was small enough to be ignored. Second, the coverage probability was close to the nominal level 0.95. Third, the inclusion of auxiliary variables in MCAR did not seem to influence the accuracy of parameter estimates and coverage probability. The use of auxiliary variables, however, boosted the power of detecting the mediation effect especially when the missing proportion was large (e.g., 40%). The findings from MAR data are similar to those from MCAR data and thus are not repeated here. However, the power of detecting mediation effects from MAR data were smaller than that from MCAR data given the same proportion of missing data.

The results for MNAR data clearly showed that when auxiliary variables were not included, parameter estimates were highly underestimated especially when the missing data proportion was large, e.g., about 67% bias with 40% missing data for the mediation effect. Correspondingly, coverage probability was highly underestimated, too. For example, with 40% of missing data, the coverage probability was only about 56%. However, with the inclusion of auxiliary variables, the parameter estimate bias dramatically decreased to less than 3% and the coverage probabilities were close to 95%. Thus, multiple imputation can be used to analyze MNAR data and recover true parameter values by including auxiliary variables that can explain missingness of the variables in the mediation model. This is because the inclusion of the auxiliary variables converts the missingness mechanism to MAR. However, this does not mean that the inclusion of auxiliary variables can always address the non-ignorable problems and more discussion on this can be found in Zhang & Wang (2013b)

Table 1.1: Bias, coverage probability, and power under MCAR

	Without Auxiliary Variables			With Auxiliary Variables		
	Bias	Coverage	Power	Bias	Coverage	Power
			MCAR			
10%	0.219	0.967	0.900	0.263	0.967	0.920
20%	-1.222	0.966	0.808	-0.593	0.963	0.845
40%	-0.716	0.946	0.531	0.112	0.950	0.615
			MAR			
10%	-0.119	0.957	0.870	-0.403	0.961	0.893
20%	-0.546	0.962	0.767	-1.940	0.958	0.791
40%	-2.932	0.960	0.511	-1.747	0.955	0.599
			MNAR			
10%	-32.633	0.831	0.800	-0.513	0.951	0.925
20%	-49.117	0.673	0.570	-2.583	0.941	0.815
40%	-66.815	0.559	0.305	-2.951	0.951	0.642

Note. We have also investigated other conditions where the sample size, population parameters, and correlation between auxiliary variables and model variables are different and observed the same patterns the results.

1.3.2 Simulation study 2. Impact of the number of imputations

One difficulty in applying multiple imputation is to decide on how many imputations are sufficient. For example, Rubin (1987) has suggested that five imputations are sufficient in the case of 50% missing data for estimating the simple mean. But Graham et al. (2007) recommend that many more imputations than what Rubin recommended should be used. Although one may always choose to use a very large number of imputations for mediation analysis with missing data, this may not be practically possible because of the amount of computational time involved.

In this simulation study, we investigate the impact of the number of imputations on point estimates and standard error estimates of mediation effects with different proportions of missing data. The same model in the first simulation study is used. The data are generated in the following way. First, two groups of 100 sets of complete data with two auxiliary variables are generated so that the correlation between the auxiliary variables and the mediation model variables is ρ =0.1 for the first group and $\rho = 0.4$ for the second group, respectively. Second, 10% and 40% of missing data are generated, respectively, for each group of the 100 sets of complete data, where the missingness is related to the auxiliary variables as in the first simulation study. Therefore, in total, we have 4 groups of 100 sets of missing data.

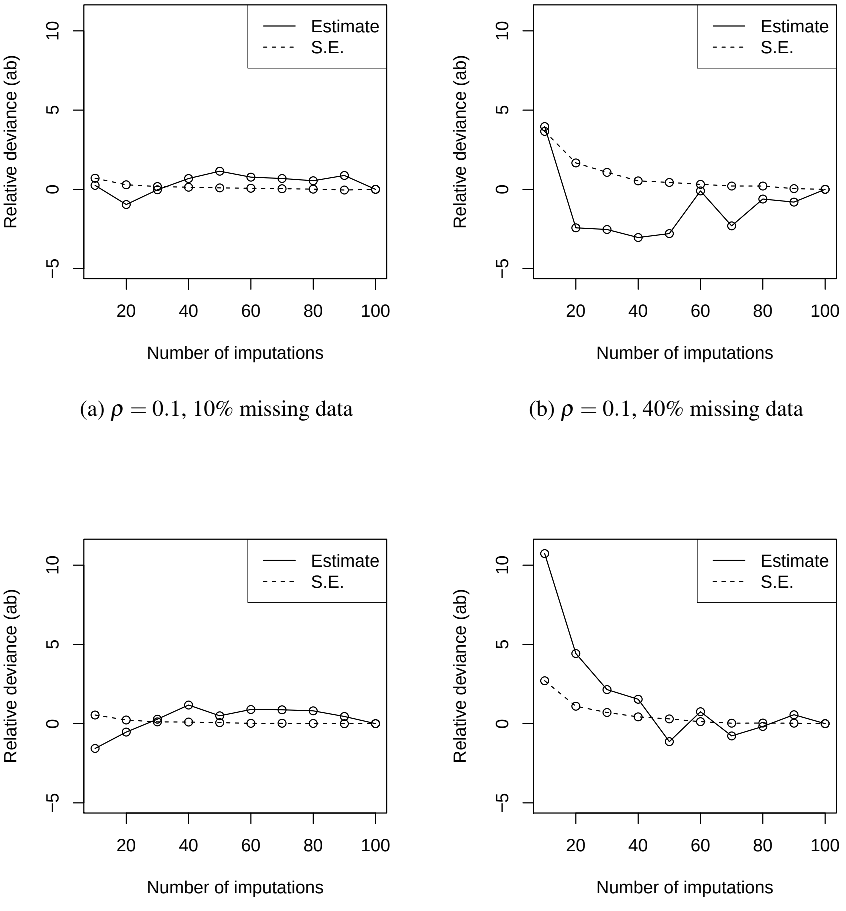
For each data set, we obtain the results from the data analysis including auxiliary variables with the number of imputations ranging from 10 to 100 at intervals of 10. Note that the overall missingness is MAR. After that, we calculate the average mediation effect and standard error estimates from the 100 sets of data. For better comparison, we calculate the relative deviance of mediation effect estimates and their standard error estimates from those estimates with 100 imputations. The relative deviance from the simulation is plotted in Figure 1.2. Since the results were based on 100 replications of simulation, the absolute difference was small as a result. Therefore, we rescaled the relative deviance by 1000 times to focus on the relative change of the deviance corresponding to the number of imputations. We have found that the analysis of individual data sets showed the similar pattern but with much larger deviance.

Comparing the results with 10% missing data in Figures 1.2a and 1.2c and the results with 40% missing data in Figures 1.2b and 1.2d, it is clear that there are more fluctuations in both mediation effect estimates and their standard errors with more missing data regardless of $\rho = 0.1$ or 0.5. Therefore, a greater number of imputations is needed with more missing data. More specifically, with 10% missing data, the parameter estimates, especially the standard estimates, seem to become stable with more than 40 or 50 imputations. With 40% missing data, however, the relative deviance of point estimates and standard error estimates does not appear stabilized until with more than 80 imputations. In our simulation study, the choice of 100 imputations appears to be adequate based on this simulation. The conclusion on the specific number of imputations here only applies to the simple mediation model. For more complex mediation analysis, more imputations might be needed.

1.4 An Empirical Example

In this section, we apply the proposed method to a real study to illustrate its application. Research has found that parental education levels can influence adolescent mathematical achievement directly and indirectly. For example, Davis-Kean (2005) showed that parental education levels are related to child academic achievement through parental beliefs and behaviors. To test a similar hypothesis, we investigate whether home environment is a mediator in the relation between mother's education and child mathematical achievement.

Data used in this example are from the National Longitudinal Survey of Youth, the 1979 cohort (NLSY79, Center for Human Resource Research, 2006). Data were collected in 1986 from $N = 475$ families on mother's education level (ME), home environment (HE), child mathematical achievement (Math), child behavior problem index (BPI), and child reading recognition and reading comprehension achievement. For the mediation analysis, mother's education is the independent variable, home environment is the mediator, and child mathematical achievement is the outcome variable. The missing data patterns and the sample size of each pattern are presented



(a) $\rho = 0.1$, 10% missing data

(b) $\rho = 0.1$, 40% missing data

(c) $\rho = 0.5$, 10% missing data

(d) $\rho = 0.5$, 40% missing data

Fig. 1.2: The impact of different numbers of imputations on the accuracy of point estimates and bootstrap standard error estimates.

in Table 1.2. In this data set, 417 families have complete data and 58 families have missing data on at least one of the two model variables: home environment and child mathematical achievement. In this study, BPI and child reading recognition and reading comprehension achievement are used as auxiliary variables because they have been found to be related to mathematical achievement in the literature (e.g., Grimm, 2008; Wu et al., 2014). In addition, it is reasonable to believe that it is more difficult to collect data from children with behavior problems and children with reading problems can have a harder time to complete tests on mathematics, which, therefore, could lead to more missing data.

Table 1.2: Missing data patterns of the empirical data set.

Pattern	ME	HE	Math	Sample size
1	O	O	O	417
2	O	X	O	36
3	O	O	X	14
4	O	X	X	8
Total				475

Note. O: observed; X: missing. ME: mother's education level; HE: home environments; Math: mathematical achievement.

In Table 1.3, the results from the empirical data analysis using the proposed method without and with the auxiliary variables are presented. The results reveal that the inclusion of the auxiliary variables only slightly changed the parameter estimates, standard errors, and the BC confidence intervals. This indicates that the auxiliary variables may not be related to the missingness in the mediation model variables.The results also show that home environment partially mediates the relationship between mother's education and child mathematical achievement because both the indirect effect *ab* and the direct effect c' are significant.

Table 1.3: Mediation effect of home environment on the relationship between mother's education and child mathematical achievement

	Without Auxiliary Variable				With Auxiliary Variable			
Parameter	Estimate	S.E.	95% BC		Estimate	S.E.	95% BC	
a	0.035	0.049	0.018	0.162	0.036	0.049	0.018	0.163
b	0.475	0.126	0.252	0.754	0.458	0.125	0.221	0.711
c'	0.134	0.191	0.071	0.611	0.134	0.188	0.072	0.609
ab	0.017	0.021	0.005	0.071	0.016	0.021	0.005	0.067
i_Y	7.953	2.047	3.530	9.825	8.045	2.025	3.778	10.006
i_M	5.330	0.556	3.949	5.641	5.327	0.558	3.945	5.646
σ^2_{eY}	4.532	0.269	4.093	5.211	4.520	0.268	4.075	5.141
σ^2_{eM}	1.660	0.061	1.545	1.789	1.660	0.061	1.542	1.790

Note. The results are based on 1000 bootstrap samples and 100 imputations. S.E.: bootstrap standard error. BC: bias-corrected confidence interval.

1.5 Software

We have developed both SAS macros and an R package bmem (Zhang & Wang, 2013a) for mediation analysis with missing data through multiple imputation and bootstrap. Instructions on how to use the SAS macros and the R package can be found on our website at http://psychstat.org/imps2014. The SAS macros are designed for the simple mediation model and have computational advantages that make them run faster than bmem. The R package bmem, however, can handle more complex mediation analysis with multiple mediators and latent variables. Both programs can utilize auxiliary variables to potentially handle MNAR data.

1.6 Discussion

In this study, we discussed how to conduct mediation analysis with missing data through multiple imputation and bootstrap. Through simulation studies, we demonstrated that the proposed method performed well for both MCAR and MAR without and with auxiliary variables. It is also shown that multiple imputation worked equally well for MNAR if auxiliary variables related to missingness were included, because the overall missingness becomes to MAR. The analysis the NLSY79 data revealed that home environment partially mediated the relationship between mother's education and child mathematical achievement.

1.6.1 Strength of the proposed method

The multiple imputation and bootstrap method for mediation analysis with missing data has several advantages. First, the idea of imputation and bootstrap is easy to understand. Second, multiple imputation has been widely implemented in both free and commercial software and thus can be extended to mediation analysis relatively easily. Third, it is natural and easy to include auxiliary variables in multiple imputation . Fourth, unlike the full information maximum likelihood method, multiple imputation does not assume a specific model when imputing data.

1.6.2 Assumptions and limitations

There are several assumptions and limitations of the current study. First, the current SAS program is based on a simple mediation model. In the future, the program should be expanded to allow more complex mediation analysis. Second, in applying multiple imputation, we have assumed that all variables are multivariate normally distributed. However, it is possible that one or more variables are not normally distributed. Third, the current mediation model only focuses on the cross-sectional data analysis. Some researchers have suggested that the time variable should be considered in mediation analysis (e.g., Cole & Maxwell, 2003; MacKinnon, 2008; Wang et al., 2009). Fourth, in dealing with MNAR data, we assume that useful auxiliary variables that can explain missingness in the mediation model variables are available. Therefore, the true missingness mechanism is actually MAR. However, sometimes the auxiliary variables may not be available. Other methods for dealing with MNAR data can be investigated in the future.

In summary, a method using multiple imputation and bootstrap for mediation analysis with missing data is introduced. Simulation results show that the method works well in dealing with missing data for mediation analysis under different missing mechanisms. Both SAS macros and an R package are provided to conduct mediation analysis with missing data, which is expected to promote the use of advanced techniques in dealing with missing data for mediation analysis in the future.

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