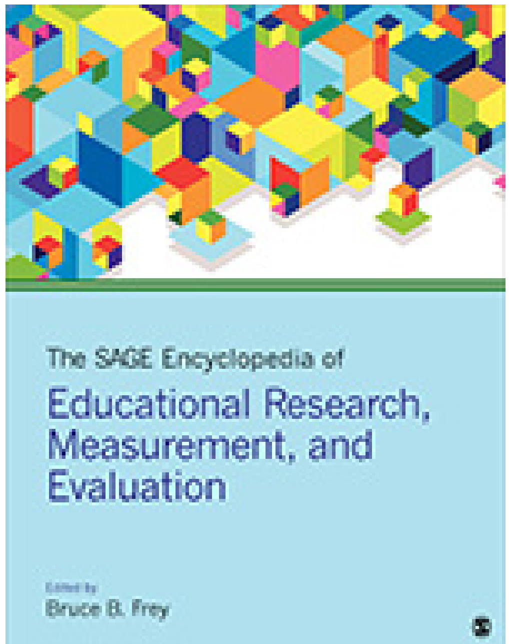

Sage Research Methods

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Author: Zhiyong Zhang
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Moments are quantitative measures of a distribution function. Formally, the *n*th moment about a value *c* of a distribution *f(x)* is defined as

$$\mu_n = E[(x - c)^n] = \left\{ \begin{array}{ll} \sum (x - c)^n f(x) & \text{Discrete distibution} \\ \int (x - c)^n f(x) dx & \text{Continuous distribution} \end{array} \right\}.$$

When $c = 0$, they are called the *raw moments*, and when *c* is set at the mean of the distributions, they are called *central moments*. The first raw moment is the mean and the first central moment is 0. For the second and higher moments, the central moments are often used. For some distributions, their moments can be flexibly obtained through their moment-generating functions. Certain distributions can be uniquely determined by a few moments. For example, a normal distribution can be determined by its first two moments. Although higher moments of a distribution can be available, the first four moments are of great interest to researchers. The remainder of this entry defines and describes those first four moments.

The first raw moment $\mu_1 = E(x) = \mu$ is the mean of a distribution and the first central moment is equal to zero. Mean is a popular measure of the central tendency of a distribution, especially for symmetric distributions.

The second central moment $\mu_2 = E[(x-\mu)^2] = \sigma^2$ is the variance of a distribution and is often denoted by σ^2. Variance is a frequently used measure of deviation from the central tendency.

The third central moment $\mu_3 = E[(x-\mu)^3]$ is related to the skewness (γ_1) of a distribution:

$$\gamma_1 = E\left[\left(\frac{x - \mu}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3}.$$

The skewness just defined is also called the third standardized moment and sometimes referred to as Pearson's moment coefficient of skewness. Skewness measures the degree of asymmetry of a distribution. For symmetric distributions such as normal and Student's *t* distributions, their skewness is 0. If the left tail of a distribution is longer than its right tail, the distribution has negative skew and the skewness is negative. If the right tail of a distribution is longer than its left tail, the distribution has positive skew and the skewness is greater than 0.

The fourth central moment $\mu_4 = E[(x-\mu)^4]$ is related to the kurtosis (γ_2) of a distribution:

$$\gamma_2 = E\left[\left(\frac{x - \mu}{\sigma}\right)^4\right] = \frac{\mu_4}{\sigma^4},$$

which is also called the fourth standardized moment. Kurtosis is associated with the tail, shoulder, and peakedness of a distribution. Generally, kurtosis increases with peakedness and decreases with flatness, while many have argued that kurtosis has as much to do with the shoulder and tails of a distribution as it does with the peakedness. The kurtosis of a normal distribution is 3. Distributions with a kurtosis less than 3 are said to be platykurtic, whereas distributions with a kurtosis greater than 3 are said to be leptokurtic. Skewness and kurtosis are often used in testing the normality of a distribution.

Table 1 summarizes the first four moments for commonly used distributions.

Table 1 Mean, Variance, Skewness, and Kurtosis for Commonly Used Distributions

Distribution	*Mean*	*Variance*	*Skewness*	*Kurtosis*
Bernoulli (p)	p	$p(1-p)$	$\frac{1-2p}{\sqrt{p(1-p)}}$	$\frac{1-3p(1-p)}{p(1-p)}$
Poisson (λ)	λ	λ	$\lambda^{-1/2}$	$\lambda^{-1}+3$
Exponential (λ)	λ^{-1}	λ^{-2}	2	9
Normal (μ,σ2)	μ	σ^2	0	3
$t(v)$	0	$v/(v-2)$	0	$(3v-6)/(v-4)$
Uniform (a,b)	$(a+b)/2$	$(b-a)^2/12$	0	1.8

See also Distributions; Kurtosis; Skewness; Variance

Zhiyong Zhang

Further Readings

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