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13

A Bayesian Discrete Dynamic System by Latent Difference Score Structural Equations Models for Multivariate Repeated Measures Data

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Longitudinal analyses by latent curves methods have become useful to model a general trajectory of behavioral responses (Rao, 1958; McArdle & Epstein, 1987; McArdle & Hamagami, 1991, 1992; Meredith & Tisak, 1990). As a methodological alternative to longitudinal data analyses, the dynamic system approach by means of difference or differential equations allows an investigation of both inter- and intravariable cause–effect relationships on the time dimension (Arminger, 1986; Beddington, Free, & Lawton, 1975; Coleman, 1968; Goldberg, 1986; McArdle, 1988; Molenaar, 1985; Nesselroade, McArdle, Aggen, & Meyers, 2001; Nesselroade & Molenaar, 1999; Tuma & Hanna, 1986; Sheinerman, 1996). In a similar vein, McArdle and Nesselroade (1994) introduced the use of latent difference scores on longitudinal factor scores derived from the multivariate longitudinal structural equation modeling (SEM) (see also McArdle, 1988). In subsequent works, McArdle and Hamagami (1995, 2001) expanded the use of difference equations applied to multiple occasions, which structurally allows dynamic interpretations. Structural latent difference score models are specifically designed to accommodate interindividual variability of initial conditions and the rate of change of the dynamic system model among different people (Nesselroade, 1991). Hamagami and McArdle (2001) demonstrated that dynamical parameters of structured latent difference score models were accurately recovered by traditional SEM under a variety of missing data situations by Monte Carlo simulations (Little & Rubin, 1987; McArdle, 1994; Schafer, 1997). Both deterministic and stochastic parameters of the dynamics system were correctly recovered using full information maximum likelihood estimation (Lange, Westlake, & Spence, 1976) using Mx program (Neale, Boker, Xie, & Maes, 2003).

In several researches mainly in quantitative social science, the Bayesian method (BE) was used to investigate multivariate latent variable models. Several investigations of Bayesian factor analyses were initially reported (Martin & McDonald, 1975; Bartholomew, 1981, 1991; Press & Shigematu, 1989). More prior researches also adopted the Bayesian approach to analyze the confirmatory factor model and robust factor models (Ansari & Jedidi, 2000; Ansari, Jedidi, & Dube, 2002; Hayashi & Sen, 2002; Hayashi & Yuan, 2003; Lee & Press, 1998).

Several previous researches on new computational algorithms have demonstrated how BEs could be used to investigate complex statistical models (Gelman, Carlin, Stern, & Rubin, 1996; Gilks, Richardson, & Spiegelhalter, 1996; Congdon, 2001, 2003). In the BE, derivation of the posterior distribution of model parameters is necessarily of utmost concern. For a simple model such as the simple regression model, it is feasible to analytically derive posterior parameter distributions (see Silva, 1996). However, analytical derivation of posterior distribution of highly complex models (e.g., Zeger & Karim, 1991) involving multiple model parameters is practically intractable due to the fact that it mathematically involves high-dimensional multiple integrals. The Gibbs sampling algorithm was introduced to circumvent this difficulty (Geman & Geman, 1984). Congdon (2001, 2003) demonstrated that Bayesian analyses are no longer limited to only simple statistical models by means of the newly developed Bayesian algorithm.

Also, the general focus of Bayesian researches was shifted to structural equation models that involve multivariate structures, latent variables, and simultaneous equations (see Arminger & Muthén, 1998; Fornell & Rust, 1989; Jedidi & Ansari, 2001; Jedidi, Ramaswamy, & DeSarbo, 1996; Lee & Song, 2003, 2004; Lee, Song, & Poon, 2004; Rupp, Dey, & Zumbo, 2004; Scheines, Hoijtink, & Boomsma, 1999; Song & Lee, 2004). Also, the Gibbs sampling algorithm allows complex nonlinear structural equation models (see Arminger et al., 1998; Lee et al., 2003, 2004).

A detailed discussion of the latent difference score model and the bivariate latent difference score model was presented in McArdle (2001) and McArdle et al. (2001). As a synopsis, we provide only a basic definition of these models. Here we focus on the Bayesian estimation (BE) to analyze these models since these models have been previously investigated by the SEM approach only. In the first part of this chapter, we apply both the Bayesian approach and traditional SEM to univariate longitudinal repeated measures data. This portion of the presentation includes univariate analyses of longitudinal data from a published example on the Wechsler Intelligence Scale for Children (McArdle & Epstein, 1987). We also employ Monte Carlo simulations to examine aspects of both estimation techniques, and we compare the results obtained by these two different estimation methods. In the second part, we examine the same problems for the more complex case of bivariate discrete dynamic systems using latent difference scores.

13.1 BE METHODS

For the estimation of the latent difference score model, the previous studies used an SEM-based approach (McArdle et al., 2001, 2004; McArdle, 2001). The estimation of the model parameters can be achieved using the standard SEM software (e.g., LISREL, Mplus, Mx). We propose the BE method to estimate dynamic parameters for the latent difference score model. The Bayesian method is an approach for statistical inference in which all the uncertain parameters are interpreted in terms of Bayesian subjective probability as opposed to Fisherian or frequentist's logical (or objective epistemic) probability. The Bayesian approach starts with the formulation of a model (M) that represents our research interests. Then we characterize a *prior distribution* $p(\theta)$ to the unknown parameters of the model, which represents our beliefs, or already established prior information about the parameters before collecting data. After conducting a study and obtaining empirical data $(y \mid \theta; M)$, we apply the Bayes rule to derive the posterior distribution ($p(\theta \mid y; M)$), for these unknown model parameters, which updates the degree of belief in the light of new empirical data. Let information from the data be expressed as $f(y \mid \theta; M)$, which is also the likelihood function in terms of the traditional SEM approach. Then the posterior of the unknown parameters can be expressed as probability density functions of the prior multiplied by the likelihood functions given by empirical data and model parameters. Mathematically, the posterior distribution function is expressed as

$$p(\theta \mid y; M) = \frac{p(\theta) f(y \mid \theta; M)}{\int f(y \mid \theta; M) d\theta} \propto p(\theta) f(y \theta; M).$$

Although in theory the prior distribution can be any form of density distributions, we will choose a conjugate prior distribution for inferential convenience. Conjugate distributions mean that the prior and the posterior distributions come from the same family of density distributions, usually the exponential family (Lee, 2004; Silva, 1996). If we had no prior information or we do not want to include the prior information by choice, we could specify a noninformative prior like a normal distribution with a large variance (e.g., 10e + 6).

All the inferences of the parameters are based on the posterior distribution. For example, the point estimation of θ is obtained as the average of all instances of parameter estimates, that is,

$$\hat{\theta} = E(\theta) = \int \theta \, p(\theta \mid y; M) d\theta.$$

With the increase of the number of the parameters, the analytical solution of the integration above would become impractical. The Gibbs sampling can circumvent this difficulty. Gibbs sampling is an algorithm to generate a sequence of samples from the joint distribution of two or more variables (or model parameters). The purpose of such a sequence is to approximate the multidimensional joint distribution, or to sequentially solve the formulaic complexity of multiple integrals so as to derive expected values of model parameters. The Gibbs sampling is an example of a Markov chain Monte Carlo (MCMC) algorithm. The Gibbs sampling is applicable when the joint distribution of a group of parameters cannot be expressed explicitly, but the conditional distribution of each parameter is known. The Gibbs sampling algorithm is a piecemeal process to generate an instance from the distribution of each parameter, conditional on the algorithmically updated values of the other parameters. Geman and Geman (1984) showed that the joint and marginal distributions that were derived from the MCMC sequences would converge at an exponential rate to the posterior joint and marginal distributions of parameters. The Gibbs sampling is particularly well adapted to sampling the posterior distribution of a Bayesian network, since Bayesian networks are typically specified as a collection of conditional distributions (see Gelman et al., 1995; Gilks et al., 1996; Congdon, 2003).

For the latent difference score model, we choose a noninformative prior for each parameter and estimate the model parameters using the program WinBUGS (Spiegelhalter, Thomas, & Best, 1999). We have divided the following section into two parts. The first part includes analyses of the univariate difference score models, and the second part includes analyses of the bivariate difference score models. Within each univariate and bivariate modeling framework, first we conduct a Monte Carlo simulation study in order to evaluate the performance of the BE method. Data based on both the univariate and the bivariate models are generated according to a set of preset population parameters. These parameters are means and variances of level and slope scores, residual variances, self-feedback parameters, coupling parameters, and covariances among latent level and slope scores. We set the sample size to 200, which is comparable to the size of the Wechsler Intelligence Scale for Children (WISC) data that sampled 204 children (Osborne & Suddick, 1972). Six repeated observations are generated for each subject. For each model, 50 samples are generated. The parameter estimations from the 50 samples are used to evaluate the empirical distribution of the parameters.

We also evaluate the ability of estimation methods to analyze the truncated data, which by design removes a portion of data reflecting missing measurement occasions. For the generated data, we insert the missing occasions by deleting all the observations for the third and fifth occasions. The pattern of missing occasions corresponds to the empirical WISC data (Osborne et al., 1972). The incomplete data are analyzed by both SEM and Bayesian methods.

Second, as an application to empirical data, the WISC data that were used in McArdle (1987, 2001) are applied to both the Gibbs sampling and the SEM estimation methods to examine whether or not Bayesian and SEM methods produce the same conclusions.

13.2 PART I: FITTING A UNIVARIATE LATENT DIFFERENCE SCORE MODEL

13.2.1 Univariate LDS Model

Let us assume that variable Y is repeatedly observed over time ($t = 1$ to T) on a sample of subjects ($n = 1$ to N). Also, we assume that a manifest score ($Y[t]_n$) is the sum of a true score ($g[t]_n$) and an unaccounted score such as measurement errors ($ey[t]_n$):

$$Y[t]_n = g[t]_n + ey[t]_n. \tag{13.1}$$

We next define change scores as

$$\Delta g[t]_n = g[t]_n - g[t-1]_n \quad \text{and} \quad g[t]_n = g[t-1]_n + \Delta g[t]_n. \tag{13.2}$$

This means that $\Delta g[t]$ is a *latent difference score* between two successive occasions. We apply a specific definition to a latent difference score at time t as

$$\Delta g[t]_n = g_{sn} + \beta g[t-1]_n. \tag{13.3}$$

This representation simply means that a difference score between time t and time $t-1(\Delta g[t]_n)$ is determined as the sum of two terms: a self-feedback effect ($\beta g[t-1]_n$), and a linear constant effect (g_{sn}). We then algebraically manipulate a difference equation, a current true score, and a preceding true score. As a result, we express a system that has current true scores as dependent variables and immediate past scores as predictors,

$$\begin{aligned} g[t]_n &= g[t-1]_n + \Delta g[t]_n. \\ &= (1+\beta)g[t-1]_n + g_{sn}. \end{aligned} \tag{13.4}$$

This true score dynamical model is then perturbed by a residual term. Thus, observed scores are described as

$$\begin{aligned} Y[t]_n &= g[t]_n + ey[t]_n \\ &= g[t-1]_n + \Delta g[t]_n + ey[t]_n \\ &= (1+\beta)g[t-1]_n + g_{sn} + ey[t]_n. \end{aligned} \tag{13.5}$$

In order to account for sources of interindividual differences (i.e., Nesse roade, 1991) on Y, the level and linear slope components are defined at second level as

$$\begin{aligned} g_{0,n} &= \mu_{y0} + d_{g0,n}, \\ g_{s,n} &= \mu_{ys} + d_{gs,n}, \end{aligned} \qquad (13.$$

where μ_{y0} and μ_{ys} are the means of latent level and slope variables, whi $d_{g0,n}$ and $d_{gs,n}$ are individuals' deviation scores from the means of late level and slope scores.

These two latent variables are distributed as multivariate normal; that i

$$\begin{pmatrix} g_s \\ g_0 \end{pmatrix} \sim MN\left[\begin{pmatrix} \mu_{gs} \\ \mu_{g0} \end{pmatrix}, \begin{pmatrix} \phi^2_{gs} & \\ \phi_{gsg0} & \phi^2_{g0} \end{pmatrix}\right]. \qquad (13.$$

The path diagram in Figure 13.1 represents this model. In this pat diagram, squares represent observed variables, while circles represe

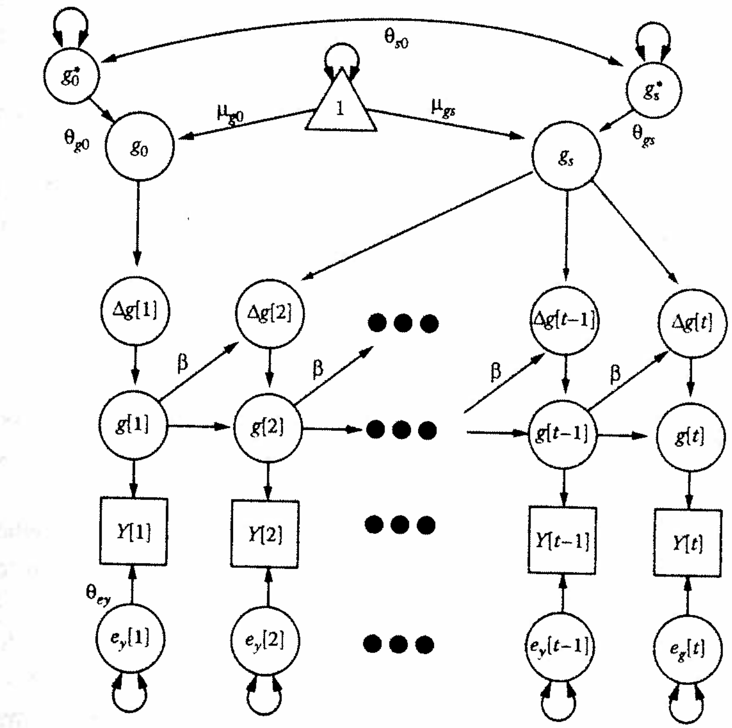

FIGURE 13.1 A univariate latent difference score model for longitudinal data.

latent variables. Single-headed arrows are deterministic parameters such as regression coefficients, or factor loadings, while a double-headed arrow represents stochastic parameters such as variance and covariance. A triangle represents a constant. Any arrow originating from the triangle represents an intercept or mean of variables pointed by the arrow. A circle labeled by $g[.]$ is a true score, a circle labeled by $\Delta g[.]$ is a latent change score, a circle labeled by g_0 is a latent level variable, and a circle labeled by g_s is a latent slope variable. A path from $g[t-1]$ to $\Delta g[t]$ is an effect of the previous true score on a current change score, which is termed as self-feedback (β). A square $Y[2]$ is pointed by $e_y[2]$ and $g[2]$. This means that $Y[2] = g[2] + ey[2]$. Similarly, $g[2]$ is pointed by $g[1]$ and $\Delta g[2]$, which is translated to $g[2] = g[1] + \Delta g[2]$. Also, $\Delta g[2]$ is pointed by $g[1]$ with a factor of β and by gs. This part of the path diagram tells us that $\Delta g[2] = \beta g[1] + g_s$.

13.2.2 Monte Carlo Results for the Univariate Model

The univariate results of Monte Carlo simulation analyses from the SEM and BE methods are summarized in Table 13.1. Table 13.1 shows the average of parameter estimates along with differences between the true values and average estimates for the univariate model based on 50 simulated samples for both complete data ($N = 200$ and six repeated measures) and incomplete data without the third and fifth measurement occasions. Table 13.2 shows the Monte Carlo standard deviations (SD_θ) and the average standard errors (MSE_θ) based on the 50 samples. These two values were calculated as

$$\mathrm{SD}_\theta = \frac{\sum_{B=1}^{50} (\theta_B - \mu_\theta)^2}{49} \quad \text{and} \quad \mathrm{MSE}_\theta = \frac{\sum_{B=1}^{50} \mathrm{SE}_{\theta,B}}{50}, \qquad (13.8)$$

where θ_B is a parameter estimate at a Monte Carlo sample, μ_θ is the average of parameter estimates over 50 samples indexed by B, and $\mathrm{SE}_{\theta,B}$ is the standard error of a parameter estimate at each Monte Carlo run.

Both SEM and Bayesian methods can accurately recover parameters of the simulated univariate model very well. When we deliberately truncated the data, overall, the parameter estimations were not as precise as those from complete data. However, the estimations were still very close to the true values. From the results, we concluded that both the SEM method and the Bayesian method equally and accurately recovered model parameters of the univariate latent difference score model.

TABLE 13.1

Average of the Estimates for Univariate Latent Difference Score Model from 50 Simulated Samples

		Complete Data				Incomplete Data			
		SEM		BE		SEM		BE	
	True	Mean	Dev.	Mean	Dev.	Mean	Dev.	Mean	Dev.
β	0.10	0.10	0	0.10	0	0.10	0	0.11	−0.01
θ^2	9.00	8.96	0.04	9.05	−0.05	9.08	−0.08	9.30	−0.30
μ_{ys}	3.00	2.97	0.03	3.05	−0.05	2.95	0.05	2.71	0.29
μ_{y0}	20.00	19.97	0.03	19.94	0.06	19.96	0.04	20.02	−0.02
ϕ_s^2	1.00	1.02	−0.02	1.05	−0.05	1.02	−0.02	0.94	0.06
ϕ_{s0}	2.00	1.99	0.01	2.13	−0.13	2.02	−0.02	1.92	0.08
ϕ_0^2	20.00	20.01	−0.01	19.98	0.02	19.81	0.19	19.81	0.19

Note: BE refers to the Bayesian estimation method; SEM refers to the structural equation modeling method; "True" indicates population parameter values; "Mean" indicates the mean of parameter estimates over 50 Monte Carlo samples; "Dev." indicates a difference between true and average estimates; for the model parameters, β refers to a self-feedback parameter of the model; θ^2 is residual variance; μ_{ys} is the mean of the linear slope; μ_{y0} is the mean of the latent level; ϕ_s^2 is variance of the latent slope; ϕ_{s0} is covariance between the latent slope and level; and ϕ_0^2 is variance of the latent level.

TABLE 13.2

Comparisons of Empirical Standard Deviation and Mean Standard Errors for the Simulated Univariate Latent Difference Score Model

	Complete				Incomplete			
	SEM		BE		SEM		BE	
	SD	MSE	SD	MSE	SD	MSE	SD	MSE
β	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01
θ^2	0.46	0.45	0.47	0.46	0.69	0.64	0.72	0.67
μ_{ys}	0.34	0.39	0.34	0.30	0.43	0.46	0.44	0.38
μ_{y0}	0.38	0.37	0.37	0.36	0.37	0.37	0.38	0.36
ϕ_s^2	0.23	0.20	0.23	0.19	0.23	0.22	0.22	0.20
ϕ_{s0}	0.56	0.56	0.58	0.53	0.62	0.61	0.63	0.58
ϕ_0^2	2.50	2.45	2.54	2.46	2.70	2.52	2.76	2.58

Note: BE: Bayesian estimation method; SEM: structural equation modeling method. SD is computed as standard deviation of parameter estimates over 50 Monte Carlo simulations; MSE is computed as average of standard errors of parameter estimates over 50 simulation samples; for the latent difference score model, β refers to a self-feedback parameter of the model; θ^2 is residual variance; μ_{ys} is the mean of the linear slope; μ_{y0} is the mean of the latent level; ϕ_s^2 is variance of the latent slope; ϕ_{s0} is covariance between the latent slope and level; and ϕ_0^2 is variance of the latent level.

TABLE 13.3

Univariate Analysis of the WISC Data Verbal Variable

		SEM		BE	
		Estimate	SE	Estimate	SE
Self-feed parameter ($v[t-1] \rightarrow v[t]$)	β	0.07	0.03	0.10	0.02
Uniqueness variance	θ^2	12.59	1.25	12.47	0.91
Mean of the slope	μ_{ys}	2.83	0.87	1.78	0.59
Mean of the level	μ_{y0}	20.15	0.55	20.39	0.39
Variance of the slope	ϕ_s^2	1.10	0.39	0.76	0.20
Covariance of the level and slope	ϕ_{s0}	1.67	1.07	0.63	0.74
Variance of the level	ϕ_0^2	20.45	3.94	20.55	2.81

Note: BE refers to the Bayesian estimation method; SEM refers to the structural equation modeling method.

Furthermore, from the SDs calculated from the estimates of parameters (SD) and the average standard errors (MSE) in Table 13.3, we could conclude: (1) overall, the MSEs were smaller than SDs, which means that the standard errors were underestimated for both methods, no matter whether missing data were present or not; (2) generally, the SEM method and the BE method could estimate the parameters with the same degree of accuracy; and; (3) when data were incomplete, the estimates were slightly less accurate than when data were complete. However, even with truncated data, all parameters were accurately estimated.

The points above can be easily grasped by inspecting the empirical density plots of the self-feedback parameter and slope mean (refer to Figure 13.2). For example, density distributions of the self-feedback parameter (β) based on the complete data showed more leptokurtic shape than that based on the incomplete data. Under both complete and incomplete simulated data, shapes of density distributions were morphometrically similar between the BE and SEM. This implies that Bayesian and SEM estimates of each simulation run are numerically similar to each other.

13.2.3 Results of the Analysis of the WISC Data

As mentioned earlier, the data structure of the WISC data included four measurement occasions only (ages 6, 7, 9, and 11 in years). Since a latent difference score model requires an equidistance between adjacent occasions, all subjects were considered as having missed the third occasion (age

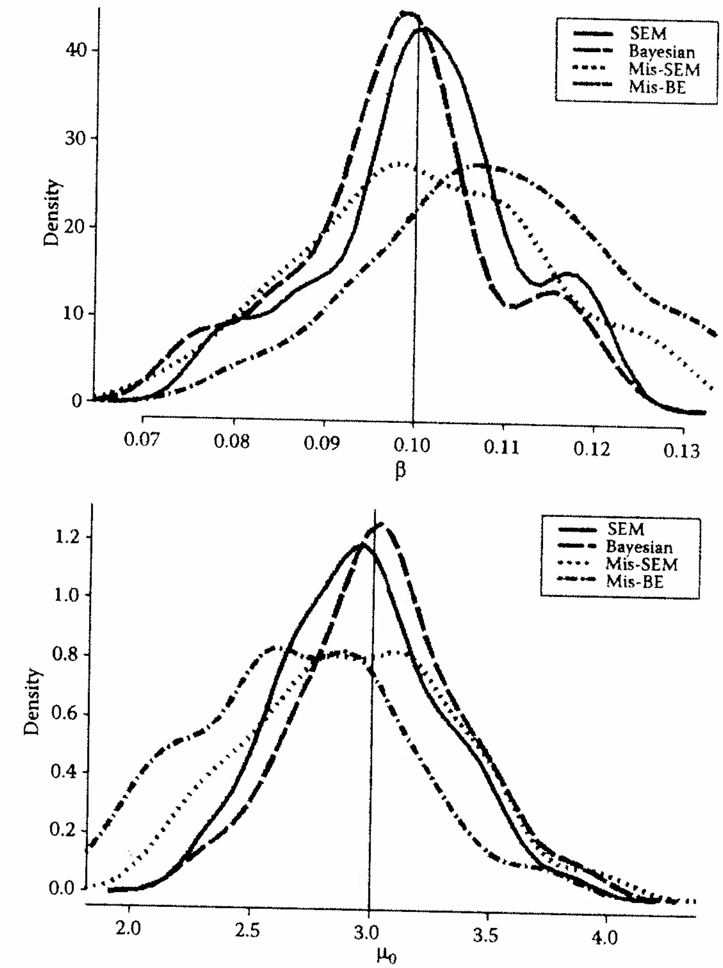

FIGURE 13.2 Density distribution plots for the posterior self-feedback parameters the posterior slope mean based on 50 Monte Carlo samples.

8) and the fifth occasion (age 10) of six repeated measures. In terms of path diagram representation, the third and fifth occasions were filled v phantom latent variables as fillers. There were no missing values in eit the verbal or performance scores for $N = 204$ subjects. For the analysi the WISC data, the results for the univariate analysis of Verbal variable presented in Table 13.3. We observed some numerical differences betw the SEM and Bayesian estimates. However, given the standard errors estimates, the SEM estimates were within the credibility region of Bayes

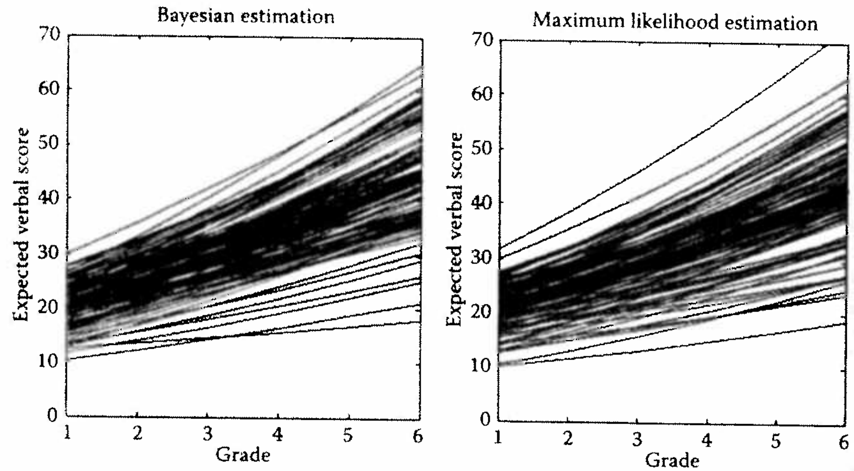

FIGURE 13.3 Expected verbal scores based on the univariate latent difference score model for the WISC verbal scores.

estimates and vice versa. Therefore, all the estimates were not statistically different when comparing between the SEM and Bayesian results. Based on the numerical results, the change scores for the verbal score were positive ($\Delta v[t] = 0.07v[t-1] + 2.83$ for BE, while $\Delta v[t] = 0.10v[t-1] + 1.79$ for SEM estimation). This implies that the higher the previous score, the higher the change score. For example, using Bayesian estimates, if the previous score was 50, then the change score would be 6.33. If the previous score was 80, the change score would be inflated to 8.43. Expected trajectories were produced in Figure 13.3. A curve in the heavy solid line was the average curve, while other curves were expected trajectories based on variance and covariance of the latent level and slope scores.

13.3 PART II: FITTING A BIVARIATE DIFFERENCE SCORE MODEL

In bivariate dynamic systems, we are interested in examining how two change processes proceed and react concurrently, and so longitudinal causal inter-relations between two change processes can be examined. A dynamic effect of one variable on a change score of another variable is coined a *coupling* effect. In an algebraic form, a bivariate difference score model is expressed as

$$\Delta g[t]_n = \beta_g g[t-1]_n + g_{sn} + \gamma_{gf} f[t-1]_n \tag{13.9}$$

for the g process, and

$$\Delta f[t]_n = \beta_f f[t-1]_n + f_{sn} + \gamma_{fg} g[t-1]_n \tag{13.10}$$

for the f process. Most importantly, γ_{gf} is a coupling effect from f to Δg, and γ_{fg} is a coupling effect from g to Δf. So what do these simultaneous change scores tell us? Change scores of one variable are influenced not only by the previous score of its own variable but also by the previous score of another variable in addition to a linear slope score. These six elements concurrently feed into characteristics of growth and trajectories of two parallel processes.

By algebraically manipulating these change scores and the fundamental equations ($f[t]_n = f[t-1]n + \Delta f[t]_n$ and $g[t]_n = g[t-1]_n + \Delta g[t]_n$), we can express the true score as a dependent variable. When an error term is added to each process, we derive simultaneous equations of the observed scores for two parallel processes, that is,

$$Y[t]_n = (1+\beta_g)g[t-1]_n + g_{sn} + \gamma_{gf} f[t-1]_n + ey[t]_n \tag{13.11}$$

for the Y process, and

$$X[t]_n = (1+\beta_f)f[t-1]_n + f_{sn} + \gamma_{fg} g[t-1]_n + ex[t]_n \tag{13.12}$$

for the X process. Latent level (initial conditions, g_0 and f_0) and slope scores for the g and f processes are assumed to evince interindividual variability. Therefore,

$$\begin{aligned} g_{0,n} &= \mu_{g0} + d_{g0,n}, \\ g_{s,n} &= \mu_{gs} + d_{gs,n}, \\ f_{0,n} &= \mu_{f0} + d_{f0,n}, \\ f_{s,n} &= \mu_{fs} + d_{fs,n}. \end{aligned} \tag{13.13}$$

Equation 13.10 shows that latent level and slope scores are defined by their mean (μ_{g0}, μ_{gs}, μ_{f0}, and μ_{fs}) and deviations from respective means

(d_{g0}, d_{gs}, d_{f0}, and d_{fs}). These four latent variables are distributed as multivariate-normal, that is,

$$\begin{pmatrix} g_0 \\ g_s \\ f_0 \\ f_s \end{pmatrix} \sim MN \left[\begin{pmatrix} \mu_{g0} \\ \mu_{gs} \\ \mu_{f0} \\ \mu_{fs} \end{pmatrix}, \begin{pmatrix} \phi_{g0}^2 & & \text{symmetric} & \\ \phi_{g0gs} & \phi_{gs}^2 & & \\ \phi_{g0f0} & \phi_{gsf0} & \phi_{f0}^2 & \\ \phi_{g0fs} & \phi_{gsfs} & \phi_{f0fs} & \phi_{fs}^2 \end{pmatrix} \right]. \quad (13.14)$$

In the above symmetric matrix, variance and covariance terms among the level and slope variables are represented by ϕ. The path diagram for the bivariate difference score model is shown in Figure 13.4. In this path diagram, a path from a circle representing a true score ($g[.]$ and $f[.]$) point to a circle representing a latent change variable ($\Delta f[.]$ and $\Delta g[.]$) is a dynamical parameter of interest. Specifically, a structural path originating from a circle labeled by $g[.]$ to a circle labeled by $\Delta f[.]$ represents a dynamical coupling from g to a latent change in f, while that from $f[.]$ to $\Delta g[.]$ represents a coupling from f to a latent change in g. Similarly, a path originating from a circle labeled by $g[.]$ to a circle labeled by $\Delta g[.]$ represents a self-feedback of the g process. All self-feedback parameters within a variable are equal

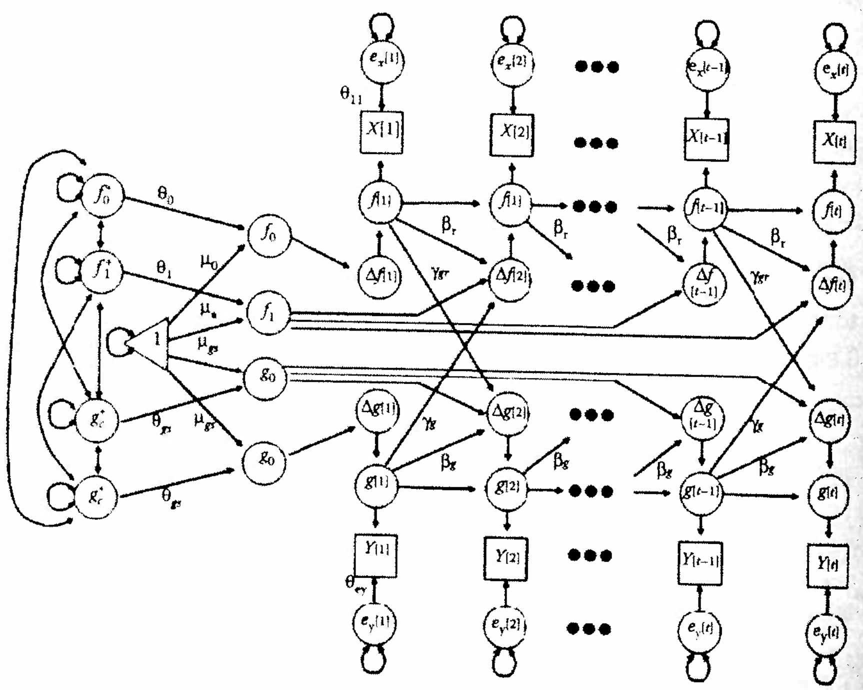

FIGURE 13.4 Path diagram for a bivariate latent difference score model.

over time. Also, we set equality constraints on coupling parameters
time within a variable. Thus, we only estimate four separate dynam
parameters, that is, two self-feedback and two coupling parameters.

13.3.1 Monte Carlo Results for the Bivariate Model

The parameter estimates for the bivariate difference score model base SEM and the MCMC method are summarized in Table 13.4. The a age standard errors and the SD of the estimated parameters summar over 50 Monte Carlo samples are given in Table 13.5. Parallel to find from the univariate longitudinal analyses, both the SEM and the Baye methods demonstrated consensus in accuracy of recovering preset par eters. Most critical dynamic parameters (i.e., $\beta_g = -0.5$, $\gamma_{gf} = -0$ $\beta_f = -0.5$, $\gamma_{fg} = 0.25$, $\mu_{gs} = 47.5$, and $\mu_{gs} = 32.5$) were recovered high accuracy for both SEM and Bayesian approaches. For example, ave dynamic systems based on the complete simulation data were estimat

$$\begin{cases} \Delta g[t] = -0.5g[t-1] - 0.25f[t-1] + 47.55 \\ \Delta f[t] = -0.5f[t-1] - 0.25g[t-1] + 32.39 \end{cases} \quad (13.$$

based on the SEM approach, and

$$\begin{cases} \Delta g[t] = -0.5g[t-1] - 0.25f[t-1] + 47.40 \\ \Delta f[t] = -0.5f[t-1] - 0.25g[t-1] + 32.33 \end{cases} \quad (13.$$

based on BE.

These derived difference equations were very close to the preset dynan system. This indicates that both Bayesian and SEM analyses were abl recover the eigen system that uniquely characterizes multivariate dynam For the incomplete data analyses, both SEM and Gibbs sampling meth recovered the true dynamics and numerically the average dynamic syste were obtained as

$$\begin{cases} \Delta g[t] = -0.52g[t-1] - 0.24f[t-1] + 47.59 \\ \Delta f[t] = -0.51f[t-1] - 0.27g[t-1] + 32.30 \end{cases} \quad (13.$$

based on the SEM method, and

$$\begin{cases} \Delta g[t] = -0.5g[t-1] - 0.25f[t-1] + 47.47 \\ \Delta f[t] = -0.5f[t-1] - 0.25g[t-1] + 32.29 \end{cases} \quad (13.$$

based on the Gibbs sampling method.

TABLE 13.4

Parameter Estimates for the Bivariate Latent Difference Score Model

		Complete Data				Incomplete Data			
		SEM		BE		SEM		BE	
Parameter	True	Mean	Dev.	Mean	Dev.	Mean	Dev.	Mean	Dev.
β_g	−0.50	−0.50	0	−0.50	0	−0.52	0.02	−0.50	0
γ_{gf}	−0.25	−0.25	0	−0.25	0	−0.24	−0.01	−0.25	0
β_f	−0.50	−0.50	0	−0.50	0	−0.51	0.01	−0.50	0
γ_{fg}	0.25	0.25	0	0.25	0	0.27	−0.02	0.25	0
μ_{gs}	47.50	47.55	−0.05	47.40	0.10	47.59	−0.09	47.47	0.03
μ_{g0}	5.00	5.04	−0.04	5.12	−0.12	5.05	−0.05	5.10	−0.10
μ_{fs}	32.50	32.39	0.11	32.33	0.17	32.30	0.20	32.29	0.21
μ_{f0}	5.00	5.00	0	5.04	−0.04	5.02	−0.02	5.04	−0.04
θ^2_{ey}	25.00	25.09	−0.09	25.66	−0.66	25.37	−0.37	26.19	−1.19
θ^2_{ex}	25.00	24.95	0.05	25.62	−0.62	24.58	0.42	25.75	−0.75
ϕ^2_{gs}	6.25	5.92	0.33	5.77	0.48	6.44	−0.19	5.67	0.58
ϕ_{gsg0}	6.25	6.20	0.05	6.36	−0.11	6.53	−0.28	6.35	−0.10
ϕ^2_{g0}	25.00	25.22	−0.22	23.14	1.86	24.92	0.08	23.25	1.75
ϕ_{gsfs}	−1.25	−1.16	−0.09	−1.08	−0.17	−1.78	0.53	−1.06	−0.19
ϕ_{g0fs}	2.50	2.33	0.17	2.32	0.18	1.93	0.57	2.19	0.31
ϕ^2_{fs}	6.25	6.25	0	6.31	−0.06	7.05	−0.80	6.24	0.01
ϕ_{gsf0}	2.50	2.22	0.28	1.95	0.55	2.10	0.40	1.85	0.65
ϕ_{g0f0}	−5.00	−4.95	−0.05	−5.19	0.19	−5.09	0.09	−4.81	−0.19
ϕ_{fsf0}	6.25	6.32	−0.07	6.66	−0.41	6.75	−0.50	6.80	−0.55
ϕ^2_{f0}	25.00	25.16	−0.16	23.00	2.00	25.37	−0.37	22.84	2.16

Note: BE: Bayesian estimation method; SEM: structural equation modeling method. "True" indicates population parameter values; "Mean" indicates the mean of parameter estimates over 50 Monte Carlo samples; "Dev." indicates a difference between true and average estimates. β_g is self-feedback of the verbal score; γ_{gf} is a coupling from the performance to the verbal score; β_f is a self-feedback of the performance score; γ_{fg} is a coupling from the verbal to performance scores; μ_{gs} is the mean of the verbal slope; μ_{g0} is the mean of the verbal level; μ_{fs} is the mean of the performance slope; μ_{g0} is the mean of the performance level; θ^2_{ey} is the uniqueness of the verbal scores; θ^2_{ex} is the uniqueness of the performance scores; $\phi^2_{..}$ is variance of either level or slope, while $\phi_{..}$ is covariance of level and slope components, where a subscript ".." represents either gs (the verbal slope), $g0$ (the verbal level), fs (the performance slope), or f_0 (the performance level).

Parallel to the findings from univariate analyses, results based on the complete data exhibited smaller average standard errors and SD of parameter estimates over 50 Monte Carlo simulations than those based on the incomplete data.

TABLE 13.5

Comparisons of Empirical Standard Deviation and Mean Standard Errors for the Simulated Bivariate Latent Difference Score Model

	Complete				Incomplete			
	SEM		BE		SEM		BE	
	SD	MSE	SD	MSE	SD	MSE	SD	MSE
β_g	0.02	0.02	0.02	0.02	0.13	0.02	0.02	0.02
γ_{gf}	0.01	0.01	0.01	0.01	0.07	0.01	0.01	0.01
β_f	0.01	0.01	0.01	0.01	0.07	0.01	0.01	0.02
γ_{fg}	0.02	0.02	0.02	0.02	0.15	0.03	0.02	0.02
μ_{gs}	0.42	0.45	0.43	0.46	0.64	0.50	0.50	0.51
μ_{g0}	0.51	0.50	0.51	0.49	0.51	0.50	0.51	0.50
μ_{fs}	0.51	0.45	0.50	0.46	0.62	0.51	0.51	0.47
μ_0	0.57	0.50	0.59	0.49	0.60	0.50	0.60	0.49
θ^2_{ey}	1.24	1.25	1.21	1.27	3.04	1.77	2.20	1.83
θ^2_{ex}	1.04	1.24	1.08	1.26	1.28	1.73	1.31	1.74
ϕ^2_{gs}	0.81	0.86	0.80	0.85	3.88	1.23	1.00	1.04
ϕ_{gsg0}	1.72	1.40	1.46	1.39	2.48	1.60	1.47	1.48
ϕ^2_{g0}	4.32	4.58	4.34	4.34	5.07	4.73	4.99	4.48
ϕ_{gsfs}	0.63	0.63	0.61	0.62	4.39	0.96	0.65	0.75
ϕ_{g0fs}	1.42	1.40	1.37	1.36	2.95	1.64	1.57	1.48
ϕ^2_{fs}	0.91	0.91	0.89	0.95	4.69	1.33	1.01	1.08
ϕ_{gsf0}	1.21	1.39	1.20	1.37	2.24	1.56	1.37	1.53
ϕ_{g0f0}	3.48	3.19	3.61	3.10	3.45	3.22	3.67	3.17
ϕ_{fsf0}	1.60	1.43	1.32	1.42	2.12	1.62	1.43	1.52
ϕ^2_{f0}	4.25	4.55	4.49	4.47	4.40	4.72	4.78	4.68

Note: See note for Table 13.2.

The density plots of the coupling parameters (γ_{gf} and γ_{fg}) and self-feedback parameters (β_g and β_f) were depicted in Figures 13.5 and 13.6, respectively. For the bivariate models, the sampling distribution of parameter estimates exhibited similar shapes between the SEM and Gibbs sampling methods regardless of whether data were complete or not.

13.3.2 Results for the Bivariate WISC

For the bivariate analysis of Verbal and Nonverbal scores, the model comparisons are summarized in Table 13.6. The results for the full difference

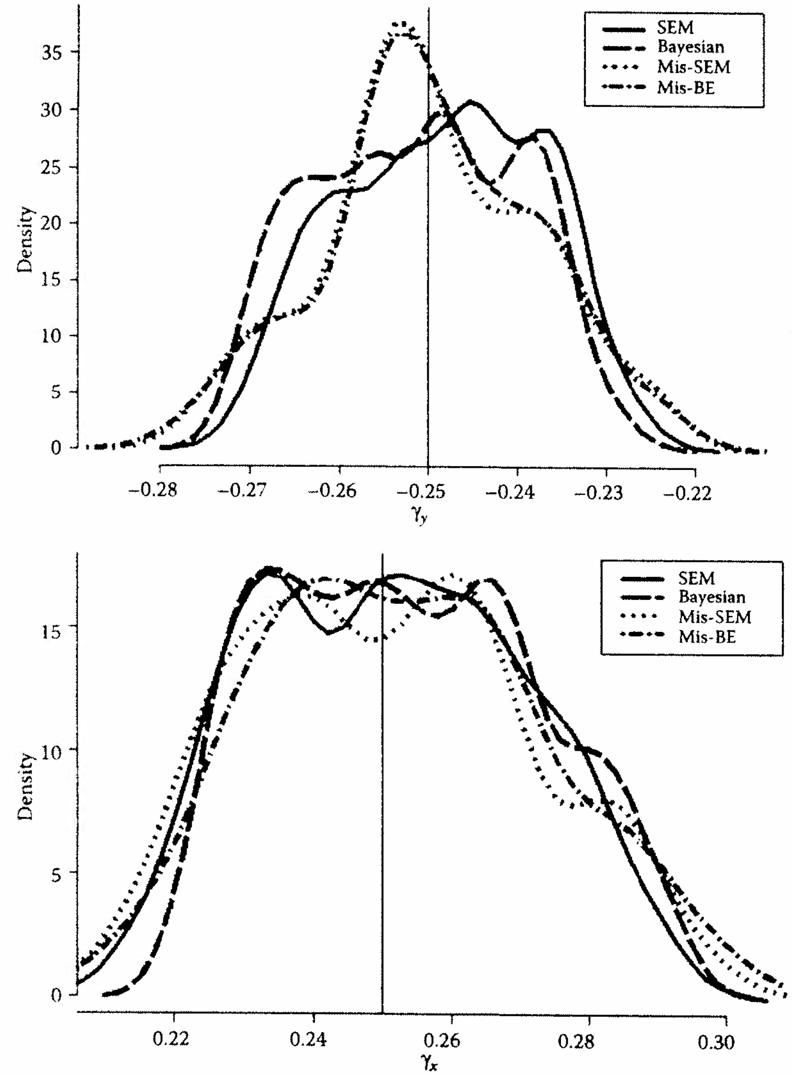

FIGURE 13.5 Density plots for the posterior coupling parameters of the bivariate latent difference score model based on 50 Monte Carlo samples.

score model are given in Table 13.7 (verbal is g and performance is f). We fitted four alternative bivariate models to the WISC data. Fit comparisons among these four models were used to examine whether one variable influenced a change in the other, or vice versa. The first model was the full model with both coupling parameters in the dynamic system. The second

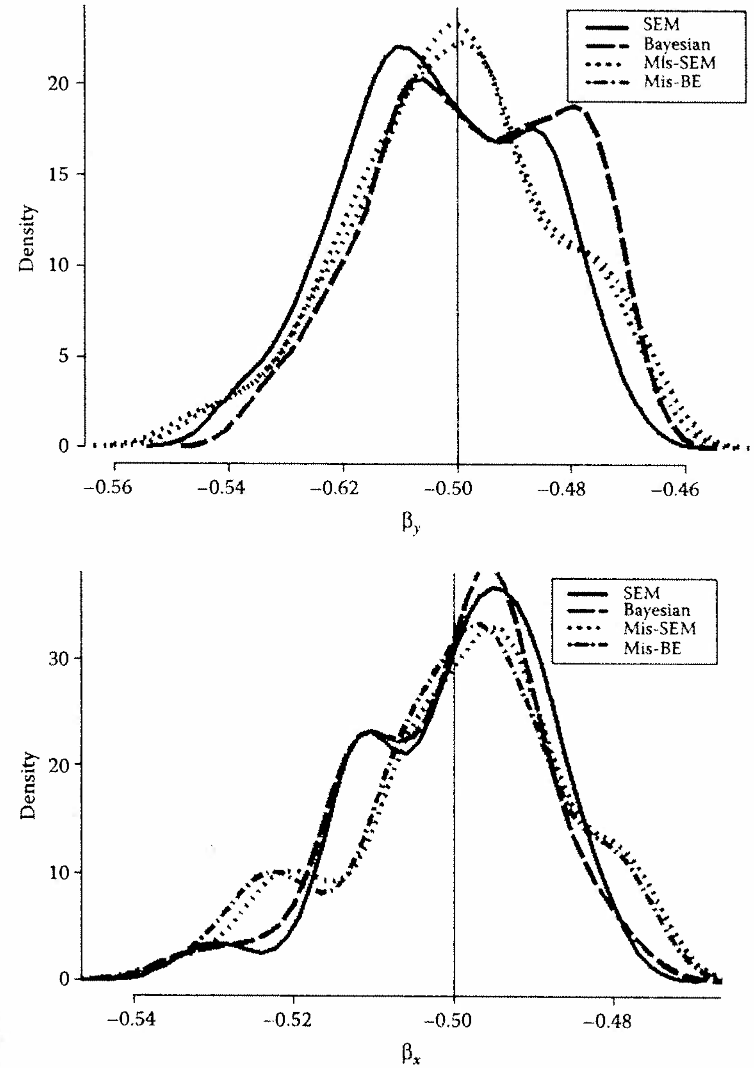

FIGURE 13.6 Density plots for the posterior self-feedback parameters of the bivariate latent difference score model based on 50 Monte Carlo samples.

model removed the coupling parameter from the Verbal to the Nonverbal, while the third model removed the coupling parameter from the Nonverbal to the Verbal. The last model removed both coupling effects from the system. For Bayesian analyses the deviance information criterion (DIC) was used to compare alternative models. A DIC difference of greater than 10 signals a substantial fit difference in which the lower valued DIC indicates a

TABLE 13.6

Model Fit Statistics

	SEM				WinBUGS	
	χ^2	AIC	BIC	RMSEA	pD	DIC
Full model	56	5200	5248	0.12	469	9665*
No nonverb on verb	65	5201	5251	0.13	477	9665*
No verb on nonverb	82	5219	5267	0.15	458	9692
No coupling	86	5220	5267	0.15	461	9699

Note: *The DIC for the full model is slightly smaller than that for the no nonverb on verb model; $\chi^2 = N^* - 2^* \log(L)$, where L refers to a likelihood function value based on ML estimation and N is the sample size; AIC $= -2^* \log(L) + 2^* p$, where p is the number of model parameters; BIC $= -2^* \log(L) + p^* \log(N)$; RMSEA $= \sqrt{\max\left[\left(\frac{2^* L}{\text{df}} - \frac{1}{N}\right), 0\right]}$ is a root mean square error of approximation. In this equation, df refers to degree of freedom; $pD = E[D(\theta)] - D[E(\theta)]$, where $D(\theta) = -2\log(L(y \mid \theta))$, a deviance index at parameter θ given by data y, and $E[]$ is an expectation operator; pD is called the effective number of parameters of the model. It is defined as the difference between the expected value of deviance ($E[D(\theta)]$) and a deviance value at the expected value of model parameters ($D[E(\theta)]$); DIC $= pd + E[D(\theta)]$ is deviance information criterion. Smaller DIC indicates better fit to data.

better fit. Based on BE, the model with a coupling effect from Nonverbal to Verbal removed was the best fit model, whereas SEM estimation indicated that the full model was the best fitted to the WISC data. A difference in χ^2 between the full model and any other model was statistically significant. Based on the parameter estimates of the bivariate full difference score model where both coupling parameters were assumed to exist, the derived dynamic system was described as

$$\begin{cases} \Delta v[t] = 0.44v[t-1] - 0.25p[t-1] + 0.27 \\ \Delta p[t] = -0.37p[t-1] + 0.32v[t-1] + 9.11 \end{cases} \quad \text{for SEM approach,} \tag{13.19}$$

$$\begin{cases} \Delta v[t] = 0.40v[t-1] - 0.20p[t-1] - 0.02 \\ \Delta p[t] = -0.31p[t-1] + 0.27v[t-1] + 9.01 \end{cases} \quad \text{for the BE.} \tag{13.20}$$

There were numerical differences in parameter estimates between the SEM and Gibbs sampling methods. However, each parameter estimate obtained via the SEM approach was located within the Bayesian credibility interval of an estimate obtained by the Gibbs sampling method. The mean of the slope factor of the Verbal process ($\mu_{gs} = 0.27$ by SEM and $\mu_{gs} = -0.02$ by BE) was seemingly numerically different. However, in both cases, they were found to be statistically not significant. Figure 13.7 shows expected latent curves of both Verbal and Nonverbal scores based on the full bivariate latent difference score model for $N = 100$. Two subfigures in the upper row are the expected latent curves derived from the Bayesian parameter estimates; the other two subfigures in the lower row were the expected curves based on SEM parameter estimates. By a visual inspection, there was similarity in the average latent curves of both Verbal and Nonverbal scores between BE and SEM estimations.

TABLE 13.7

Parameter Estimation for the Bivariate Model of WISC Data (Fixing the Variances of Slopes)

Parameters of the Model		SEM		BE	
		Est.	SE	Est.	SE
Self-feedback of the verbal	β_g	0.44	0.07	0.39	0.04
Coupling from the performance to the verbal	γ_{gf}	−0.25	0.04	−0.20	0.03
Self-feedback of the performance	β_f	−0.37	0.06	−0.31	0.06
Coupling from the verb to performance	γ_{fg}	0.32	0.09	0.27	0.08
Mean of the slope of the verbal score	μ_{gs}	0.27	1.13	−0.02	0.59
Mean of the level of the verbal score	μ_{g0}	20.05	0.55	20.20	0.40
Mean of the slope of the performance	μ_{fs}	9.11	1.25	9.01	0.91
Mean of the level of the performance	μ_{f0}	18.07	0.81	18.22	0.62
Uniqueness of the verbal score	θ^2_{ey}	11.89	1.17	11.71	0.83
Uniqueness of the performance	θ^2_{ex}	22.70	2.21	22.41	1.43
Variance of the verbal slope	ϕ^2_{gs}	2.31	0.00	2.31	0.58
Covariance of the v level and v slope	ϕ_{gsg0}	−0.99	1.40	−1.80	1.01
Variance of the verbal level	ϕ^2_{g0}	21.54	3.81	22.93	2.89
Covariance of the v slope and p slope	ϕ_{gsfs}	3.63	0.38	3.62	0.83
Covariance of the v level and p slope	ϕ_{g0fs}	3.43	2.14	1.85	1.55
Variance of the performance slope	ϕ^2_{fs}	7.99	0.00	7.99	1.91
Covariance of the v slope and p level	ϕ_{gsf0}	4.80	2.00	3.84	1.60
Covariance of the v level and p level	ϕ_{g0f0}	24.90	4.82	28.61	3.83
Covariance of the p slope and p level	ϕ_{fsf0}	14.58	2.70	14.25	2.39
Variance of the performance level	ϕ^2_{f0}	46.39	8.83	55.97	7.13

13.4 DISCUSSION

Previously, several researches demonstrated that the Bayesian technique could be used for analyses of a latent variable model such as static factor

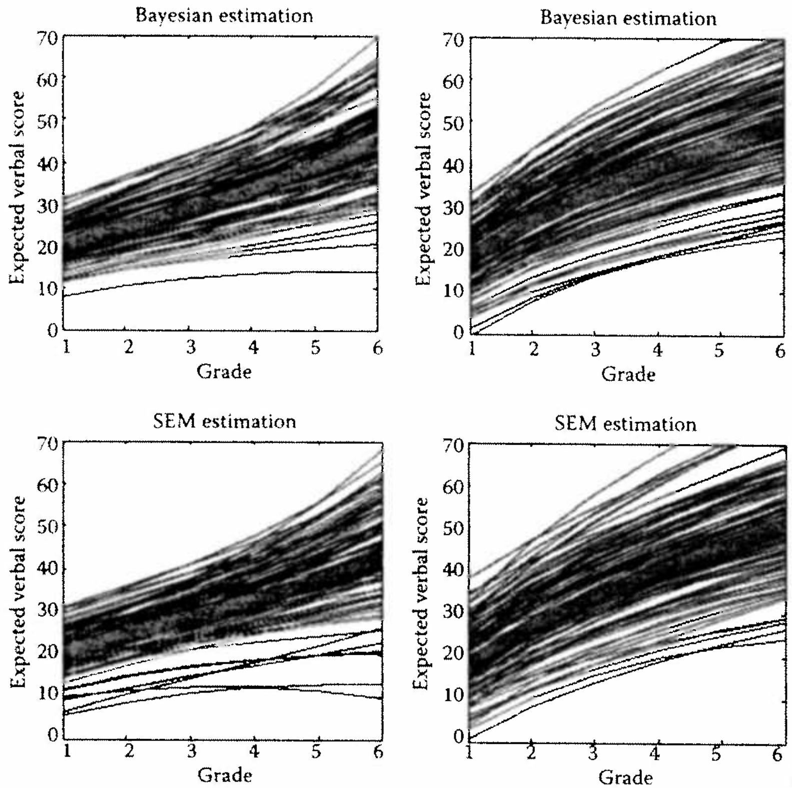

FIGURE 13.7 Expected verbal and performance latent growth curves based on the bivariate latent difference score models for the WISC verbal and performance scores.

models (e.g., Ansari et al., 2002; Bartholomew, 1981). One aspect of our research was focused on a longitudinal structural equation model using multivariate repeated measures. In particular, our research focused on comparison of the SEM and BE methods to numerically identify the discrete dynamic system models involving multiple subjects, multivariate repeated measures with interindividual and intraindividual variability of key latent variables. For this purpose, we adopted latent difference score models (McArdle, 2001; McArdle et al., 2001) as a dynamic system example. With the use of the Monte Carlo simulation study, we demonstrated that both SEM and BE methods were viable alternatives to numerically identify a system of latent difference score equations. Even with six repeated measures data format, both the SEM and the Bayesian approaches were able to accurately recover the population parameters for the preset dynamics system. In addition, we also demonstrated that even deliberately removing one-third of data from the simulation data, we were still able to correctly identify the dynamic system using both the SEM and the Gibbs sampling methods.

Our research justifies the use of the Gibbs sampling to estimate parameters for the complex latent variable longitudinal structural equation models where the number of latent variables far exceeds the number of observed variables in the model. Basically we concur with Arminger et al. (1998), Jedidi et al. (2001), and Lee et al. (2003, 2004) that the BE is a useful technique to model structural equation models. Prior to this research project, we used the Gibbs sampling method to investigate latent growth curves (Zhang, Hamagami, Wang, Nessleroade, & Grimm, 2007). We found that both SEM and Gibbs sampling produced almost identical results when noninformative priors were chosen. The findings of this study were consistent with previous findings on SEM and BE—the Bayesian algorithms and MLE-based SEM algorithms will yield the same numerical conclusions in cases where no prior information for parameters is provided. However, under Bayesian theory, if prior information is available and it is weighted heavily, the Bayesian approach can provide parameter estimates and goodness-of-fit indices that are more appropriate than standard MLE-based SEM estimates. With WinBUGS program (Spiegelhalter et al., 1999), it is now possible to analyze a variety of complex structured latent models, including the multilevel SEM models described as multiple group SEM by McArdle and Hamagami (1996). Recent books by Congdon (2001, 2003) and Gilks et al. (1996) illustrate that the Gibbs sampling method using WinBUGS program can be applied to models including categorical variable models, multilevel models, factor models, and SEM as examples for data analyses. One asset of WinBUGS is a script syntax that eliminates repetitive model statements that usually have to be spelled out in the traditional SEM software. As the number of repeated measures increases, repetitive statements make SEM programming laborious and tedious, while script syntaxes of WinBUGS simplify programming.

With interfacing the WinBUGS and SAS programs, BEs are now easily performed within the SAS computational environment. This interface allows reformat of SAS data into WinBUGS-readable forms as well as generation of initial values of the model parameters to be used for the WinBUGS. After the MCMC estimation algorithm stabilizes Bayes estimates, results are read back into the SAS environment to further examine the results.

Details of the procedures are described in Zhang, McArdle, Wang, and Hamagami (2008).

Finally, we recognize an ongoing controversy between Fisherian and Bayesian interpretation of statistical probability. This research is neither intended to advocate one method over the other, nor is this paper intended to resolve such a complex controversial issue. Instead, we can report an interesting finding about the issue of when the Bayesian approach could be beneficial or could become problematic.

13.4.1 When does Bayesian Modeling Work?

There are several factors to consider when it comes to the effectiveness of BE. One factor is whether or not the prior knowledge about the causal relationship is correct. Another factor is whether or not the study collects data from the true population. If the sample does not represent the target population, any estimation method, Bayesian or not, would not help reach the correct conclusion about the causal relationship. To answer the question of when BE helps, simulation analyses were conducted for the simple regression model. The model represents a simple regression in which a criterion variable, Y is predicted by a stimulus X. There are three parameters: (1) an intercept, (2) a regression coefficient, and (3) a residual variance. The study manipulated (1) the sample size ranging from $N = 20$ to $N = 1000$, (2) the status of the prior information, correct or incorrect, and (3) the characteristics of the sample, representing the true or false population. Numerical details of the Monte Carlo simulation are not presented here since this is not a primary focus of our paper. Instead, the major findings are presented.

13.4.2 Scenario 1: Using Noninformative Prior and the Sample from the Correct Population

Since the prior is noninformative, the data from the sample predominantly determines estimation of parameters. If the data were sampled from the correct population, the parameter estimates could be close to the population parameters. Generally the noninformative prior would lead to correct estimation of the population parameter. The larger the sample size, the more precise the estimation of the population parameters. This is the case that both Bayesian and non-Bayesian would come to the congruent conclusion where the inference about a conclusion is totally data dependent.

13.4.3 Scenario 2: Using Noninformative Prior and the Sample from the False Population

When the sample comes from the wrong population and the noninformative prior is used for BE, parameter estimates on average would converge near the parameters that are true to the false population. Therefore, this scenario would lead to a wrong inference about the causal relationship. The wrong sample is detrimental to any causal study. This is the case that both Bayesian and non-Bayesian would come to the wrong conclusion about the causal relationship. The wrong conclusion is totally manufactured by the wrong data.

13.4.4 Scenario 3: Using the Wrong Prior but the Sample from the Correct Population

When the sample is drawn from the true population but the wrong informative prior is used for BE, the parameter estimates are generally incorrect and lead to a wrong inference about a causal relationship. In this case, even the sample size over $N = 500$ will not help resolve an inaccurate conclusion about the causal relationship. This is the case that maximum likelihood estimation would result in the correct conclusion.

13.4.5 Scenario 4: Using the Strong Wrong Prior and the Sample from the False Population

When the sample is drawn from the wrong population and the wrong informative prior is used, parameter estimates are way off from the true parameters. This scenario would lead to a wrong inference about a causal relationship. It is apparent that the wrong sample and wrong prior knowledge exacerbate inaccuracy in the inference of a causal relationship. This is the case that we would not want to face since BE does not help.

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13.4.6 Scenario 5: Using the Correct Prior and the Sample from the True Population

When the sample is drawn from the true population and the strong inf
mative prior is set at the exact parameter value for the correct populati
parameter estimates are generally trusted even with the small sample. T
is the ideal scenario where subjective belief and sampling process mutu
support each other and inference about the causal relationship from
situation is reliable. This is the case that we the audience would want to
since BE would augment the prior knowledge and subjective belief ab
the causal inference with the updated information.

13.4.7 Scenario 6: Using the Strong Correct Prior and the Sample from the False Population

When the sample is drawn from the wrong population but the informa
prior is tapping the correct parameters expressive of the true populat
inference about a causal relationship depends on the sample size. With
small sample sizes, that is, weak influence of the empirical data, the cor
prior information on the target parameter will help in tapping the
rect value and Bayesian estimates are in the neighborhood of the cor
parameter values. With the large sample from the wrong population,
influence of the large amount of wrong data will dominate over the in
ence of the correct prior. Thus, this situation leads to a wrong inferenc
a causal relationship.

In conclusion, our main goal of this paper was to show that Bayesian
SEM could be used to fit data to a longitudinal structural equation mod
dynamical nature. Deriving a final conclusion about a causal relation
is an extremely complicated process since the final decision is influence
so many factors such as model misspecification, missampling from a wr
population, nonrandom sampling, nontrustworthy prior knowledge
misinformation about the causal relationship, timing of observations w
critical time of the growth process might be completely missed, amou
empirical observations, and other unpreventable circumstances. All t
factors are likely to contribute to generating biases in parameter estim
to some degree whether or not either Bayesian or SEM estimation is op
For this reason, this paper was not intended to show our predilectio
one methodology over the other. Any quantitative method conduci

aking a correct decision about a causal relationship should be employed
r furthering any scientific knowledge.

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14

Longitudinal Mediation Analysis of Training Intervention Effects

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14.1 INTRODUCTION

Intervention research constitutes a broad research category, encompassing a large number of experimental, psychological, and medical research designs. While this type of research can build on rather simple designs and few variables, multivariate longitudinal designs allow for tests of more complex treatment assignments and models, as well as mediation. The focus of this chapter is on models and methods for evaluating longitudinal intervention effects in the presence of mediation: that is, the methods for analyzing the longitudinal impact of an intervention or treatment when that impact is mediated by one or more other variables.

We explore the concept of mediation and its application to longitudinal intervention research. First, we review the history and basic ideas of mediation analysis, including developments in longitudinal mediation analysis. Second, we present a variation of the longitudinal mediation model (Cole & Maxwell, 2003) for intervention and training research, highlighting the features of the nonrepeated training interventions, the repeatedly measured mediation and output variables, and the estimation methods of the model. Finally, we apply this new model to data from the Advanced Cognitive Training for Independent and Vital Elderly (ACTIVE) study (Jobe et al., 2001) to demonstrate the capabilities of the model.