Eight Simulation Result Tables for Bayesian GMM With Missing Data

Zhenqiu Laura Lu
University of Notre Dame
Zhiyong Zhang
University of Notre Dame
Gitta Lubke
University of Notre Dame

Table 1: Results with N=1500, Equal Class Probabilities, and MNAR

		Parameter	True [a]	Est.avg [b]	Bias.rel [c]	SD.emp [d]	SD.avg [e]	HPD.cvr [f]
Growth Curve Parameters	Class 1	$\beta_1[1]$	25	25.020	0.001	0.106	0.106	0.93
		$\beta_1[2]$	1.1	1.095	-0.004	0.024	0.025	0.96
		$\Psi_1[11]$	6	6.000	0.000	0.429	0.424	0.94
		$\Psi_1[22]$	0.2	0.208	0.041	0.024	0.024	0.96
		$\Psi_1[12]$	-0.8	-0.802	0.003	0.088	0.087	0.95
		ϕ_1	2	1.989	-0.005	0.076	0.070	0.93
	Class 2	$\beta_2[1]$	16	15.956	-0.003	0.222	0.207	0.96
		$\beta_2[2]$	1.3	1.298	-0.002	0.053	0.054	0.98
		$\Psi_2[11]$	16	15.783	-0.014	1.379	1.494	0.97
		$\Psi_2[22]$	0.4	0.385	-0.038	0.087	0.098	0.97
		$\Psi_2[12]$	0.8	0.831	0.039	0.232	0.278	0.97
		ϕ_2	13	13.005	0.000	0.456	0.454	0.94
Probit Parameters	Class	φ_0	-0.5	-0.491	-0.018	0.058	0.057	0.94
		φ_1	0.5	0.493	-0.014	0.038	0.040	0.94
	Wave 1	γ^*_{01}	-1	-1.032	0.032	0.109	0.120	0.98
		γ^*_{11}	0.3	0.337	0.124	0.127	0.142	0.98
		γ_{x1}	-1.05	-1.071	0.020	0.077	0.080	0.98
	Wave 2	γ^*_{02}	-1	-1.031	0.031	0.098	0.107	0.96
		γ^*_{12}	0.34	0.366	0.077	0.108	0.125	0.98
		γ_{x2}	-0.75	-0.761	0.014	0.064	0.064	0.96
	Wave 3	γ^*_{03}	-1	-1.003	0.003	0.094	0.098	0.96
		γ^*_{13}	0.37	0.375	0.015	0.106	0.113	0.98
		γ_{x3}	-0.55	-0.565	0.027	0.050	0.055	0.98
	Wave 4	γ^*_{04}	-1	-1.002	0.002	0.098	0.093	0.92
		γ^*_{14}	0.41	0.414	0.009	0.115	0.104	0.94
		γ_{x4}	-0.41	-0.419	0.021	0.048	0.049	0.96
	Wave 5	γ^*_{05}	-1	-1.012	0.012	0.097	0.088	0.94
		γ^*_{15}	0.44	0.445	0.011	0.097	0.096	0.95
		γ_{x5}	-0.28	-0.267	-0.045	0.049	0.045	0.91

[a] The true value of the parameter.
[b] The average Bayesian point estimates defined in Eq (25).
[c] The relative deviation for the averaged estimate from its true value defined in Eq (26).
[d] The empirical S.D. of the estimates defined in Eq (27).
[e] The average Bayesian S.D. defined in Eq (28).
[f] The coverage of 95% HPD credible interval defined in Eq (29).

Table 2: Results with N=1000, Equal Class Probabilities, and MNAR

		Parameter	True	Est.avg	Bias.rel	SD.emp	SD.avg	HPD.cvr
Growth Curve Parameters	Class 1	$\beta_1[1]$	25	25.006	0.000	0.135	0.131	0.96
		$\beta_1[2]$	1.1	1.096	-0.003	0.032	0.032	0.96
		$\Psi_1[11]$	6	6.034	0.006	0.541	0.525	0.97
		$\Psi_1[22]$	0.2	0.216	0.080	0.030	0.030	0.94
		$\Psi_1[12]$	-0.8	-0.818	0.023	0.117	0.108	0.93
		ϕ_1	2	1.997	-0.002	0.090	0.085	0.94
	Class 2	$\beta_2[1]$	16	16.002	0.000	0.223	0.254	0.96
		$\beta_2[2]$	1.3	1.304	0.003	0.073	0.067	0.93
		$\Psi_2[11]$	16	15.882	-0.007	1.892	1.827	0.95
		$\Psi_2[22]$	0.4	0.435	0.088	0.100	0.123	0.98
		$\Psi_2[12]$	0.8	0.820	0.025	0.304	0.345	0.98
		ϕ_2	13	13.005	0.000	0.524	0.558	0.97
Probit Parameters	Class	φ_0	-0.5	-0.502	0.003	0.071	0.071	0.97
		φ_1	0.5	0.504	0.008	0.046	0.050	0.96
	Wave 1	γ^*_{01}	-1	-1.015	0.015	0.157	0.151	0.93
		γ^*_{11}	0.3	0.324	0.079	0.187	0.179	0.94
		γ_{x1}	-1.05	-1.096	0.044	0.104	0.100	0.98
	Wave 2	γ^*_{02}	-1	-1.020	0.020	0.127	0.132	0.95
		γ^*_{12}	0.34	0.352	0.036	0.153	0.154	0.96
		γ_{x2}	-0.75	-0.759	0.012	0.092	0.079	0.94
	Wave 3	γ^*_{03}	-1	-0.979	-0.021	0.126	0.121	0.94
		γ^*_{13}	0.37	0.353	-0.045	0.151	0.139	0.93
		γ_{x3}	-0.55	-0.579	0.053	0.070	0.068	0.91
	Wave 4	γ^*_{04}	-1	-1.010	0.010	0.127	0.115	0.94
		γ^*_{14}	0.41	0.420	0.023	0.147	0.129	0.92
		γ_{x4}	-0.41	-0.423	0.033	0.064	0.061	0.96
	Wave 5	γ^*_{05}	-1	-1.011	0.011	0.112	0.109	0.94
		γ^*_{15}	0.44	0.445	0.011	0.115	0.120	0.96
		γ_{x5}	-0.28	-0.280	0.000	0.049	0.056	0.96

Note: With the same notations as those in Table 1 above.

Table 3: Results with N=1500, Equal Class Probabilities, and MCAR

		Parameter	True	Est.avg	Bias.rel	SD.emp	SD.avg	HPD.cvr
Growth Curve Parameters	Class 1	$\beta_1[1]$	25	25.026	0.001	0.112	0.110	0.94
		$\beta_1[2]$	1.1	1.093	-0.006	0.026	0.027	0.95
		$\Psi_1[11]$	6	5.991	-0.001	0.489	0.445	0.94
		$\Psi_1[22]$	0.2	0.209	0.045	0.026	0.027	0.95
		$\Psi_1[12]$	-0.8	-0.804	0.005	0.102	0.094	0.93
		ϕ_1	2	1.987	-0.007	0.074	0.078	0.96
	Class 2	$\beta_2[1]$	16	15.959	-0.003	0.219	0.206	0.90
		$\beta_2[2]$	1.3	1.299	-0.001	0.051	0.053	0.96
		$\Psi_2[11]$	16	15.794	-0.013	1.428	1.473	0.95
		$\Psi_2[22]$	0.4	0.383	-0.043	0.081	0.095	0.95
		$\Psi_2[12]$	0.8	0.829	0.036	0.257	0.271	0.96
		ϕ_2	13	13.015	0.001	0.468	0.441	0.95
Probit Parameters	Class	φ_0	-0.5	-0.493	-0.014	0.057	0.056	0.93
		φ_1	0.5	0.493	-0.013	0.038	0.040	0.93
	Wave 1	γ^*_{01}	-1	-1.001	0.001	0.080	0.083	0.94
		γ^*_{11}	0	0.000	0.000	0.089	0.092	0.95
		γ_{x1}	0	-0.005	-0.005	0.041	0.042	0.94
	Wave 2	γ^*_{02}	-1	-0.999	-0.001	0.090	0.083	0.92
		γ^*_{12}	0	-0.007	-0.007	0.096	0.092	0.94
		γ_{x2}	0	0.001	0.001	0.039	0.042	0.93
	Wave 3	γ^*_{03}	-1	-1.009	0.009	0.091	0.083	0.92
		γ^*_{13}	0	0.000	0.000	0.097	0.092	0.94
		γ_{x3}	0	0.001	0.001	0.047	0.042	0.90
	Wave 4	γ^*_{04}	-1	-0.990	-0.010	0.076	0.083	0.93
		γ^*_{14}	0	-0.018	-0.018	0.087	0.092	0.97
		γ_{x4}	0	-0.004	-0.004	0.036	0.042	0.98
	Wave 5	γ^*_{05}	-1	-0.992	-0.008	0.09	0.083	0.92
		γ^*_{15}	0	-0.005	-0.005	0.093	0.091	0.96
		γ_{x5}	0	-0.001	-0.001	0.042	0.042	0.94

Note: With the same notations as those in Table 1 above.

Table 4: Results with N=1000, Equal Class Probabilities, and MCAR

		Parameter	True	Est.avg	Bias.rel	SD.emp	SD.avg	HPD.cvr
Growth Curve Parameters	Class 1	$\beta_1[1]$	25	24.996	0.000	0.138	0.135	0.98
		$\beta_1[2]$	1.1	1.098	-0.001	0.032	0.033	0.93
		$\Psi_1[11]$	6	6.023	0.004	0.567	0.552	0.95
		$\Psi_1[22]$	0.2	0.218	0.092	0.031	0.033	0.94
		$\Psi_1[12]$	-0.8	-0.814	0.018	0.115	0.116	0.96
		ϕ_1	2	1.994	-0.003	0.096	0.095	0.95
	Class 2	$\beta_2[1]$	16	15.991	-0.001	0.225	0.253	0.98
		$\beta_2[2]$	1.3	1.300	0.000	0.068	0.065	0.91
		$\Psi_2[11]$	16	15.766	-0.015	1.885	1.804	0.91
		$\Psi_2[22]$	0.4	0.425	0.064	0.090	0.118	0.98
		$\Psi_2[12]$	0.8	0.841	0.051	0.322	0.334	0.94
		ϕ_2	13	12.984	-0.001	0.562	0.542	0.94
Probit Parameters	Class	φ_0	-0.5	-0.496	-0.008	0.076	0.070	0.93
		φ_1	0.5	0.503	0.005	0.049	0.050	0.96
	Wave 1	γ^*_{01}	-1	-1.006	0.006	0.109	0.102	0.96
		γ^*_{11}	0	-0.001	-0.001	0.120	0.113	0.94
		γ_{x1}	0	0.002	0.002	0.058	0.052	0.93
	Wave 2	γ^*_{02}	-1	-1.001	0.001	0.094	0.103	0.96
		γ^*_{12}	0	-0.014	-0.014	0.105	0.113	0.96
		γ_{x2}	0	-0.002	-0.002	0.054	0.053	0.95
	Wave 3	γ^*_{03}	-1	-1.011	0.011	0.096	0.103	0.96
		γ^*_{13}	0	0.006	0.006	0.100	0.113	0.96
		γ_{x3}	0	0.006	0.006	0.055	0.052	0.95
	Wave 4	γ^*_{04}	-1	-1.002	0.002	0.104	0.102	0.97
		γ^*_{14}	0	-0.007	-0.007	0.111	0.113	0.95
		γ_{x4}	0	0.000	0.000	0.053	0.052	0.93
	Wave 5	γ^*_{05}	-1	-1.006	0.006	0.095	0.103	0.98
		γ^*_{15}	0	0.015	0.015	0.104	0.113	0.95
		γ_{x5}	0	-0.002	-0.002	0.049	0.052	0.97

Note: With the same notations as those in Table 1 above.

Table 5: Results with N=1500, Unequal Class Probabilities, and MNAR

		Parameter	True	Est.avg	Bias.rel	SD.emp	SD.avg	HPD.cvr
Growth Curve Parameters	Class 1	$\beta_1[1]$	25	25.008	0.000	0.116	0.136	0.97
		$\beta_1[2]$	1.1	1.097	-0.002	0.033	0.032	0.97
		$\Psi_1[11]$	6	5.979	-0.004	0.525	0.549	0.94
		$\Psi_1[22]$	0.2	0.212	0.061	0.029	0.031	0.95
		$\Psi_1[12]$	-0.8	-0.806	0.007	0.111	0.112	0.95
		ϕ_1	2	2.003	0.001	0.090	0.090	0.97
	Class 2	$\beta_2[1]$	16	15.983	-0.001	0.162	0.175	0.97
		$\beta_2[2]$	1.3	1.294	-0.005	0.045	0.046	0.96
		$\Psi_2[11]$	16	15.913	-0.005	1.413	1.261	0.91
		$\Psi_2[22]$	0.4	0.387	-0.032	0.078	0.088	0.95
		$\Psi_2[12]$	0.8	0.811	0.013	0.249	0.241	0.92
		ϕ_2	13	13.050	0.004	0.384	0.390	0.96
Probit Parameters	Class	φ_0	-1	-1.014	0.014	0.072	0.068	0.96
		φ_1	0.5	0.508	0.016	0.047	0.043	0.94
	Wave 1	γ^*_{01}	-1	-1.007	0.007	0.174	0.176	0.96
		γ^*_{11}	0.3	0.293	-0.024	0.188	0.191	0.94
		γ_{x1}	-1.05	-1.069	0.019	0.077	0.079	0.97
	Wave 2	γ^*_{02}	-1	-1.020	0.020	0.155	0.151	0.98
		γ^*_{12}	0.34	0.371	0.091	0.161	0.161	0.97
		γ_{x2}	-0.75	-0.767	0.022	0.054	0.062	0.97
	Wave 3	γ^*_{03}	-1	-1.062	0.062	0.137	0.135	0.93
		γ^*_{13}	0.37	0.433	0.170	0.145	0.142	0.94
		γ_{x3}	-0.55	-0.539	-0.020	0.047	0.052	0.95
	Wave 4	γ^*_{04}	-1	-1.009	0.009	0.122	0.120	0.94
		γ^*_{14}	0.41	0.418	0.019	0.133	0.124	0.92
		γ_{x4}	-0.41	-0.413	0.006	0.043	0.047	0.98
	Wave 5	γ^*_{05}	-1	-1.032	0.032	0.115	0.111	0.93
		γ^*_{15}	0.44	0.472	0.073	0.125	0.113	0.90
		γ_{x5}	-0.28	-0.278	-0.008	0.045	0.043	0.92

Note: With the same notations as those in Table 1 above.

Table 6: Results with N=1000, Unequal Class Probabilities, and MNAR

		Parameter	True	Est.avg	Bias.rel	SD.emp	SD.avg	HPD.cvr
Growth Curve Parameters	Class 1	$\beta_1[1]$	25	25.016	0.001	0.162	0.167	0.96
		$\beta_1[2]$	1.1	1.096	-0.004	0.034	0.039	0.97
		$\Psi_1[11]$	6	6.046	0.008	0.712	0.679	0.92
		$\Psi_1[22]$	0.2	0.223	0.115	0.037	0.038	0.94
		$\Psi_1[12]$	-0.8	-0.823	0.029	0.146	0.139	0.91
		ϕ_1	2	1.990	-0.005	0.097	0.110	0.99
	Class 2	$\beta_2[1]$	16	15.970	-0.002	0.233	0.214	0.93
		$\beta_2[2]$	1.3	1.295	-0.004	0.057	0.057	0.93
		$\Psi_2[11]$	16	15.875	-0.008	1.597	1.545	0.94
		$\Psi_2[22]$	0.4	0.401	0.002	0.079	0.104	0.99
		$\Psi_2[12]$	0.8	0.846	0.057	0.285	0.290	0.97
		ϕ_2	13	12.997	0.000	0.418	0.473	0.98
Probit Parameters	Class	φ_0	-1	-1.008	0.008	0.080	0.083	0.96
		φ_1	0.5	0.504	0.009	0.053	0.053	0.94
	Wave 1	γ^*_{01}	-1	-1.055	0.055	0.207	0.227	1.00
		γ^*_{11}	0.3	0.341	0.137	0.217	0.245	1.00
		γ_{x1}	-1.05	-1.069	0.018	0.112	0.096	0.89
	Wave 2	γ^*_{02}	-1	-1.031	0.031	0.186	0.189	0.98
		γ^*_{12}	0.34	0.374	0.101	0.200	0.201	0.96
		γ_{x2}	-0.75	-0.752	0.003	0.070	0.075	0.96
	Wave 3	γ^*_{03}	-1	-1.016	0.016	0.171	0.162	0.92
		γ^*_{13}	0.37	0.399	0.080	0.172	0.171	0.94
		γ_{x3}	-0.55	-0.567	0.030	0.062	0.065	0.98
	Wave 4	γ^*_{04}	-1	-1.025	0.025	0.140	0.150	0.97
		γ^*_{14}	0.41	0.436	0.064	0.150	0.155	0.97
		γ_{x4}	-0.41	-0.422	0.029	0.059	0.058	0.94
	Wave 5	γ^*_{05}	-1	-1.020	0.020	0.143	0.137	0.94
		γ^*_{15}	0.44	0.461	0.047	0.145	0.139	0.92
		γ_{x5}	-0.28	-0.285	0.019	0.059	0.053	0.92

Note: With the same notations as those in Table 1 above.

Table 7: Results with N=1500, Unequal Class Probabilities, and MCAR

		Parameter	True	Est.avg	Bias.rel	SD.emp	SD.avg	HPD.cvr
Growth Curve Parameters	Class 1	$\beta_1[1]$	25	25.001	0.000	0.126	0.141	0.96
		$\beta_1[2]$	1.1	1.099	-0.001	0.035	0.034	0.94
		$\Psi_1[11]$	6	5.966	-0.006	0.574	0.582	0.96
		$\Psi_1[22]$	0.2	0.216	0.079	0.034	0.034	0.95
		$\Psi_1[12]$	-0.8	-0.806	0.007	0.129	0.122	0.93
		ϕ_1	2	1.995	-0.002	0.092	0.101	0.96
	Class 2	$\beta_2[1]$	16	15.968	-0.002	0.156	0.175	0.98
		$\beta_2[2]$	1.3	1.296	-0.003	0.047	0.045	0.95
		$\Psi_2[11]$	16	15.818	-0.011	1.311	1.258	0.94
		$\Psi_2[22]$	0.4	0.381	-0.047	0.074	0.085	0.94
		$\Psi_2[12]$	0.8	0.829	0.036	0.203	0.238	0.98
		ϕ_2	13	13.049	0.004	0.370	0.383	0.96
Probit Parameters	Class	φ_0	-1	-1.010	0.010	0.072	0.067	0.90
		φ_1	0.5	0.508	0.015	0.047	0.043	0.95
	Wave 1	γ^*_{01}	-1	-1.008	0.008	0.095	0.100	0.97
		γ^*_{11}	0	0.007	0.007	0.096	0.100	0.96
		γ_{x1}	0	0.007	0.007	0.037	0.042	0.98
	Wave 2	γ^*_{02}	-1	-0.995	-0.005	0.089	0.100	0.97
		γ^*_{12}	0	-0.009	-0.009	0.095	0.099	0.97
		γ_{x2}	0	-0.004	-0.004	0.040	0.042	0.96
	Wave 3	γ^*_{03}	-1	-0.995	-0.005	0.097	0.100	0.95
		γ^*_{13}	0	-0.007	-0.007	0.100	0.100	0.96
		γ_{x3}	0	-0.005	-0.005	0.043	0.043	0.93
	Wave 4	γ^*_{04}	-1	-0.995	-0.005	0.098	0.100	0.97
		γ^*_{14}	0	0.000	0.000	0.096	0.100	0.97
		γ_{x4}	0	-0.012	-0.012	0.047	0.042	0.95
	Wave 5	γ^*_{05}	-1	-1.009	0.009	0.094	0.100	0.96
		γ^*_{15}	0	-0.002	-0.002	0.103	0.100	0.93
		γ_{x5}	0	0.000	0.000	0.042	0.043	0.95

Note: With the same notations as those in Table 1 above.

Table 8: Results with N=1000, Unequal Class Probabilities, and MCAR

		Parameter	True	Est.avg	Bias.rel	SD.emp	SD.avg	HPD.cvr
Growth Curve Parameters	Class 1	$\beta_1[1]$	25	25.005	0.000	0.174	0.174	0.93
		$\beta_1[2]$	1.1	1.100	0.000	0.038	0.042	0.98
		$\Psi_1[11]$	6	6.120	0.020	0.760	0.727	0.92
		$\Psi_1[22]$	0.2	0.228	0.138	0.037	0.042	0.94
		$\Psi_1[12]$	-0.8	-0.838	0.047	0.146	0.152	0.98
		ϕ_1	2	1.974	-0.013	0.118	0.123	0.97
	Class 2	$\beta_2[1]$	16	15.993	0.000	0.240	0.215	0.93
		$\beta_2[2]$	1.3	1.297	-0.002	0.053	0.056	0.96
		$\Psi_2[11]$	16	15.932	-0.004	1.732	1.542	0.89
		$\Psi_2[22]$	0.4	0.401	0.003	0.080	0.102	0.97
		$\Psi_2[12]$	0.8	0.856	0.070	0.269	0.286	0.95
		ϕ_2	13	12.969	-0.002	0.477	0.464	0.97
Probit Parameters	Class	φ_0	-1	-1.007	0.007	0.090	0.082	0.92
		φ_1	0.5	0.503	0.007	0.055	0.053	0.96
	Wave 1	γ^*_{01}	-1	-1.012	0.012	0.127	0.123	0.94
		γ^*_{11}	0	0.004	0.004	0.119	0.123	0.97
		γ_{x1}	0	0.001	0.001	0.057	0.052	0.93
	Wave 2	γ^*_{02}	-1	-1.021	0.021	0.120	0.124	0.93
		γ^*_{12}	0	0.021	0.021	0.120	0.123	0.94
		γ_{x2}	0	0.002	0.002	0.050	0.052	0.95
	Wave 3	γ^*_{03}	-1	-1.017	0.017	0.137	0.123	0.91
		γ^*_{13}	0	0.012	0.012	0.133	0.122	0.92
		γ_{x3}	0	0.003	0.003	0.050	0.052	0.96
	Wave 4	γ^*_{04}	-1	-1.024	0.024	0.118	0.123	0.99
		γ^*_{14}	0	0.019	0.019	0.111	0.123	0.97
		γ_{x4}	0	0.000	0.000	0.057	0.052	0.94
	Wave 5	γ^*_{05}	-1	-1.010	0.010	0.110	0.122	0.95
		γ^*_{15}	0	0.003	0.003	0.110	0.122	0.97
		γ_{x5}	0	0.006	0.006	0.054	0.052	0.95

Note: With the same notations as those in Table 1 above.