

Table 1

Real Data Analysis with Latent-Class-Dependent Missingness, 2000 burn-in and 5000 for analysis

	Parameter	Mean	S.D.	MCs.e.	$\frac{\text{MCs.e.}}{\text{S.D.}}$	2.5pct	97.5pct	Geweke t	
Growth Curve Parameters	Class 1 ^a	$\beta_1[1]$	25.162	0.167	0.010	0.060	24.830	25.490	-1.367
		$\beta_1[2]$	1.126	0.037	0.002	0.054	1.054	1.198	1.608
		$\Psi_1[11]$	5.444	0.675	0.029	0.043	4.255	6.889	1.438
		$\Psi_1[22]$	0.184	0.031	0.002	0.065	0.126	0.247	1.165
		$\Psi_1[12]$	-0.829	0.134	0.006	0.045	-1.108	-0.584	-1.847
		ϕ_1	1.893	0.104	0.005	0.048	1.693	2.101	1.99
Probit Parameters	Class 2 ^b	$\beta_2[1]$	15.926	0.165	0.006	0.036	15.600	16.250	-1.060
		$\beta_2[2]$	1.254	0.042	0.002	0.048	1.172	1.334	-1.712
		$\Psi_2[11]$	15.869	1.109	0.052	0.047	13.800	18.190	-1.453
		$\Psi_2[22]$	0.407	0.080	0.007	0.088	0.262	0.576	-2.920
		$\Psi_2[12]$	0.799	0.211	0.015	0.071	0.374	1.202	1.424
		ϕ_2	13.284	0.355	0.016	0.045	12.610	13.990	3.485
Grade 7	Class	φ_{10}^c	-0.246	0.113	0.012	0.106	-0.451	-0.03	3.866
		φ_{11}	-0.242	0.073	0.008	0.110	-0.376	-0.097	-3.666
		γ_{01}^d	-1.407	0.193	0.021	0.109	-1.833	-1.046	-0.259
	Grade 8	γ_{11}^*	0.099	0.128	0.010	0.078	-0.142	0.353	0.658
		γ_{x1}	-0.182	0.118	0.012	0.102	-0.413	0.082	-0.219
		γ_{02}^*	-2.173	0.201	0.021	0.104	-2.598	-1.835	1.521
	Grade 9	γ_{12}^*	0.45	0.16	0.015	0.094	0.166	0.787	-1.382
		γ_{x2}	0.082	0.104	0.010	0.096	-0.119	0.286	-0.63
		γ_{03}^*	-1.363	0.139	0.014	0.101	-1.629	-1.121	-0.581
	Grade 10	γ_{13}^*	0.195	0.116	0.009	0.078	-0.031	0.419	-0.220
		γ_{x3}	-0.132	0.089	0.009	0.101	-0.308	0.030	0.737
		γ_{04}^*	-1.615	0.173	0.019	0.110	-1.929	-1.274	1.354
	Grade 11	γ_{14}^*	0.182	0.128	0.011	0.086	-0.069	0.421	-0.313
		γ_{x4}	0.029	0.099	0.010	0.101	-0.172	0.231	-1.248
		γ_{05}^*	-1.490	0.161	0.017	0.106	-1.789	-1.187	4.255
	Grade 12	γ_{15}^*	0.415	0.119	0.010	0.084	0.179	0.643	-0.883
		γ_{x5}	-0.098	0.093	0.010	0.108	-0.285	0.093	-3.859

^a The growth curve parameters for class 1. Specifically, $\beta[1]$: initial level; $\beta[2]$: slope; $\Psi[11]$: variance of initial level; $\Psi[22]$: variance of slope; $\Psi[12]$: covariance of initial level and slope; ϕ : variance of error.

^b The growth curve parameters for class 2.

^c The probit parameters of class proportion.

^d The probit parameters of missing data rate. Note that although the γ_{0t}^* and γ_{1t}^* here are different with the γ_{zt_1} and γ_{zt_2} , they are equivalent after re-parameterizing $\gamma_{0t}^* = \gamma_{zt_1}$ and $\gamma_{1t}^* = \gamma_{zt_2} - \gamma_{zt_1}$.

Table 2

A summary table of $N=500$, class proportion=50% 50%, and with MNAR missingness

	parameter	true	est. ^a	bias ^b	rl.bias ^c	se.emp ^d	se.avg ^e	MSE ^f	ci.l ^g	ci.u ^h	ci.cv ⁱ	hpd.l ^j	hpd.u ^k	hpd.cv ^l
Growth Curve Parameters	I ^m	25	25.025	0.025	0.001	0.185	0.185	0.069	24.662	25.391	0.94	24.653	25.375	0.94
	S ⁿ	1.1	1.089	-0.011	-0.01	0.047	0.046	0.004	0.995	1.177	0.96	0.997	1.178	0.97
	var(I)	6	5.922	-0.078	-0.013	0.771	0.737	1.145	4.61	7.494	0.94	4.529	7.383	0.94
	var(S)	0.2	0.227	0.027	0.134	0.038	0.042	0.004	0.152	0.317	0.91	0.147	0.311	0.94
	cov(IS)	-0.8	-0.813	-0.013	0.016	0.147	0.154	0.045	-1.14	-0.538	0.95	-1.119	-0.523	0.96
	var(e)	2	2	0	0	0.139	0.121	0.034	1.774	2.247	0.91	1.766	2.237	0.91
Class 1	I	16	15.986	-0.014	-0.001	0.35	0.362	0.253	15.284	16.701	0.96	15.273	16.683	0.96
	S	1.3	1.307	0.007	0.005	0.097	0.095	0.019	1.12	1.495	0.96	1.119	1.492	0.96
	var(I)	16	15.929	-0.071	-0.004	2.433	2.604	12.722	11.284	21.472	0.94	11.018	21.114	0.94
	var(S)	0.4	0.455	0.055	0.138	0.122	0.158	0.044	0.205	0.815	0.99	0.179	0.767	0.99
	cov(IS)	0.8	0.778	-0.022	-0.028	0.474	0.467	0.446	-0.208	1.627	0.96	-0.158	1.666	0.97
	var(e)	13	13.044	0.044	0.003	0.786	0.779	1.223	11.591	14.643	0.96	11.527	14.561	0.95
Class 2	φ_{10}	-0.5	-0.505	-0.005	0.009	0.094	0.101	0.019	-0.705	-0.31	0.96	-0.701	-0.309	0.96
	φ_{11}	0.5	0.507	0.007	0.015	0.075	0.071	0.011	0.371	0.649	0.94	0.369	0.646	0.95
	γ_{01}^*	-1	-1.005	-0.005	0.005	0.208	0.219	0.093	-1.471	-0.607	0.97	-1.435	-0.59	0.97
	γ_{11}^*	0.3	0.288	-0.012	-0.039	0.254	0.261	0.134	-0.197	0.83	0.96	-0.212	0.797	0.96
	γ_{x1}	-1.05	-1.105	-0.055	0.052	0.137	0.142	0.042	-1.393	-0.836	0.96	-1.385	-0.831	0.95
	γ_{02}^*	-1	-1.034	-0.034	0.034	0.211	0.199	0.087	-1.449	-0.67	0.93	-1.427	-0.66	0.95
Probit Parameters	γ_{12}^*	0.34	0.391	0.051	0.149	0.243	0.23	0.116	-0.038	0.865	0.95	-0.049	0.843	0.97
	γ_{x2}	-0.75	-0.785	-0.035	0.046	0.112	0.112	0.026	-1.01	-0.571	0.92	-1.005	-0.568	0.93
	γ_{03}^*	-1	-1.047	-0.047	0.047	0.169	0.177	0.062	-1.408	-0.714	0.95	-1.394	-0.706	0.95
	γ_{13}^*	0.37	0.39	0.02	0.054	0.208	0.203	0.085	0.003	0.799	0.93	-0.001	0.788	0.94
	γ_{x3}	-0.55	-0.555	-0.005	0.009	0.091	0.096	0.017	-0.747	-0.37	0.96	-0.743	-0.369	0.96
	γ_{04}^*	-1	-1.041	-0.041	0.041	0.18	0.165	0.061	-1.375	-0.727	0.94	-1.363	-0.72	0.95
Wave 1	γ_{14}^*	0.41	0.444	0.034	0.084	0.185	0.184	0.069	0.09	0.815	0.96	0.087	0.805	0.98
	γ_{x4}	-0.41	-0.409	0.001	-0.003	0.101	0.086	0.018	-0.578	-0.243	0.9	-0.576	-0.242	0.9
	γ_{05}^*	-1	-1.032	-0.032	0.032	0.148	0.156	0.047	-1.348	-0.734	0.95	-1.339	-0.731	0.95
	γ_{15}^*	0.44	0.475	0.035	0.079	0.165	0.171	0.057	0.147	0.816	0.96	0.146	0.811	0.96
	γ_{x5}	-0.28	-0.284	-0.004	0.015	0.076	0.079	0.012	-0.44	-0.13	0.95	-0.438	-0.131	0.96
Wave 2														
Wave 3														
Wave 4														
Wave 5														

^a The average of the parameter estimates across all replications.^b The average of the absolute biases across all replications.^c The average of the relative biases across all replications.^d The empirical standard errors across all replications.^e The average of the standard errors across all replications.^f The average of the mean square error, calculated by $bias^2 + se^2$, across all replications.^g The average of the lower bound of the confidence interval across all replications.^h The average of the upper bound of the confidence interval across all replications.ⁱ The average of the coverage of the confidence interval across all replications.^j The average of the lower bound of the HPD interval across all replications.^k The average of the upper bound of the HPD interval across all replications.^l The average of the coverage of the HPD interval across all replications.^m Interceptⁿ Slope

Table 3

A summary table of N=500, class proportion=50% 50%, and with MCAR missingness

	para.	true	est.	bia	rl.bia	se.emp	se.avg	MSE	CI.L	CI.U	CI.CV	HPD.L	HPD.U	HPD.CV	
Growth Curve Parameters	Class 1	I	25	25.012	0.012	0	0.183	0.19	0.07	24.64	25.385	0.93	24.63	25.371	0.93
		S	1.1	1.092	-0.008	-0.008	0.048	0.048	0.005	0.996	1.184	0.93	0.997	1.183	0.93
		var(I)	6	5.944	-0.056	-0.009	0.794	0.78	1.244	4.559	7.614	0.97	4.473	7.496	0.96
		var(S)	0.2	0.228	0.028	0.141	0.04	0.046	0.005	0.147	0.327	0.94	0.142	0.32	0.97
		cov(IS)	-0.8	-0.809	-0.009	0.011	0.164	0.165	0.054	-1.161	-0.514	0.95	-1.137	-0.497	0.95
		var(e)	2	1.999	-0.001	-0.001	0.146	0.135	0.039	1.748	2.276	0.94	1.739	2.265	0.95
Growth Curve Parameters	Class 2	I	16	15.972	-0.028	-0.002	0.363	0.359	0.261	15.273	16.681	0.94	15.263	16.665	0.94
		S	1.3	1.31	0.01	0.007	0.089	0.093	0.017	1.128	1.492	0.96	1.126	1.489	0.97
		var(I)	16	15.875	-0.125	-0.008	2.274	2.559	11.752	11.31	21.319	0.97	11.025	20.938	0.97
		var(S)	0.4	0.444	0.044	0.11	0.119	0.151	0.04	0.205	0.789	0.99	0.18	0.743	0.98
		cov(IS)	0.8	0.827	0.027	0.034	0.435	0.454	0.398	-0.13	1.654	0.96	-0.082	1.691	0.95
		var(e)	13	13.047	0.047	0.004	0.822	0.754	1.243	11.638	14.592	0.92	11.578	14.518	0.92
Probit Parameters	Wave 1	φ_{10}	-0.5	-0.5	0	0.001	0.091	0.099	0.018	-0.699	-0.31	0.94	-0.694	-0.309	0.96
		φ_{11}	0.5	0.503	0.003	0.006	0.073	0.07	0.01	0.368	0.643	0.94	0.368	0.642	0.94
	Wave 2	γ_{01}^*	-1	-1.006	-0.006	0.006	0.128	0.145	0.037	-1.296	-0.726	0.97	-1.289	-0.723	0.98
		γ_{11}^*	0	0.001	0.001	0.001	0.15	0.161	0.048	-0.313	0.319	0.97	-0.311	0.316	0.96
	Wave 3	γ_{x1}	0	0.009	0.009	0.009	0.071	0.073	0.011	-0.135	0.153	0.96	-0.134	0.152	0.96
		γ_{02}^*	-1	-1.017	-0.017	0.017	0.164	0.146	0.048	-1.308	-0.735	0.91	-1.301	-0.733	0.91
Probit Parameters	Wave 4	γ_{12}^*	0	-0.011	-0.011	-0.011	0.176	0.161	0.057	-0.325	0.308	0.92	-0.325	0.304	0.93
		γ_{x2}	0	0.005	0.005	0.005	0.088	0.074	0.013	-0.14	0.15	0.89	-0.14	0.149	0.89
	Wave 5	γ_{03}^*	-1	-1.014	-0.014	0.014	0.137	0.146	0.04	-1.306	-0.734	0.96	-1.3	-0.732	0.96
		γ_{13}^*	0	0.014	0.014	0.014	0.157	0.161	0.051	-0.3	0.331	0.96	-0.3	0.327	0.97
	Wave 4	γ_{x3}	0	-0.002	-0.002	-0.002	0.072	0.074	0.011	-0.147	0.143	0.94	-0.146	0.142	0.94
		γ_{04}^*	-1	-1.022	-0.022	0.022	0.156	0.146	0.046	-1.315	-0.742	0.94	-1.308	-0.74	0.93
Probit Parameters	Wave 5	γ_{14}^*	0	0.003	0.003	0.003	0.166	0.161	0.053	-0.311	0.321	0.95	-0.307	0.321	0.95
		γ_{x4}	0	0.01	0.01	0.01	0.075	0.074	0.011	-0.134	0.156	0.96	-0.134	0.154	0.95
	Wave 5	γ_{05}^*	-1	-0.987	0.013	-0.013	0.15	0.145	0.044	-1.276	-0.706	0.93	-1.271	-0.705	0.94
		γ_{15}^*	0	-0.02	-0.02	-0.02	0.159	0.162	0.051	-0.335	0.298	0.96	-0.334	0.295	0.96
		γ_{x5}	0	-0.011	-0.011	-0.011	0.086	0.074	0.013	-0.156	0.134	0.9	-0.155	0.133	0.88

Table 4

A summary table of N=500, class proportion=30% 70%, and with MNAR missingness

Table 5

A summary table of $N=500$, class proportion=30% 70%, and with MCAR missingness

	para.	true	est.	bia	rl.bia	se.emp	se.avg	MSE	CI.L	CI.U	CI.CV	HPD.L	HPD.U	HPD.CV	
Growth Curve Parameters	Class 1	I	25	25.018	0.018	0.001	0.244	0.243	0.119	24.537	25.492	0.94	24.531	25.481	0.93
		S	1.1	1.09	-0.01	-0.009	0.051	0.059	0.006	0.974	1.203	0.97	0.975	1.203	0.97
		var(I)	6	5.992	-0.008	-0.001	0.9	1.028	1.879	4.215	8.232	0.97	4.08	8.038	0.98
		var(S)	0.2	0.232	0.032	0.162	0.045	0.058	0.006	0.135	0.36	0.99	0.126	0.347	0.99
		cov(IS)	-0.8	-0.809	-0.009	0.011	0.172	0.214	0.076	-1.272	-0.436	0.99	-1.236	-0.41	0.99
		var(e)	2	1.995	-0.005	-0.002	0.17	0.174	0.059	1.677	2.357	0.93	1.663	2.339	0.94
Growth Curve Parameters	Class 2	I	16	16.046	0.046	0.003	0.283	0.304	0.174	15.454	16.645	0.95	15.446	16.631	0.96
		S	1.3	1.292	-0.008	-0.006	0.076	0.079	0.012	1.138	1.447	0.95	1.136	1.444	0.94
		var(I)	16	15.888	-0.112	-0.007	2.051	2.173	8.937	11.949	20.458	0.99	11.746	20.187	0.99
		var(S)	0.4	0.421	0.021	0.053	0.096	0.132	0.028	0.207	0.718	1	0.186	0.683	0.99
		cov(IS)	0.8	0.845	0.045	0.056	0.339	0.39	0.271	0.026	1.558	0.97	0.067	1.589	0.97
		var(e)	13	12.956	-0.044	-0.003	0.626	0.645	0.808	11.742	14.267	0.99	11.698	14.211	0.99
Probit Parameters	Wave 1	φ_{10}	-1	-1.03	-0.03	0.03	0.121	0.119	0.03	-1.271	-0.805	0.93	-1.263	-0.801	0.94
		φ_{11}	0.5	0.52	0.02	0.039	0.08	0.076	0.013	0.374	0.671	0.93	0.373	0.668	0.92
		γ_{01}^*	-1	-1.001	-0.001	0.001	0.206	0.178	0.074	-1.358	-0.659	0.9	-1.349	-0.657	0.9
		γ_{11}^*	0	0.011	0.011	0.011	0.195	0.177	0.069	-0.332	0.365	0.95	-0.334	0.357	0.95
		γ_{x1}	0	-0.006	-0.006	-0.006	0.08	0.074	0.012	-0.151	0.14	0.93	-0.15	0.139	0.92
	Wave 2	γ_{02}^*	-1	-1.051	-0.051	0.051	0.185	0.179	0.069	-1.41	-0.707	0.95	-1.4	-0.705	0.95
		γ_{12}^*	0	0.031	0.031	0.031	0.195	0.178	0.07	-0.311	0.385	0.93	-0.313	0.377	0.94
		γ_{z2}	0	0.016	0.016	0.016	0.072	0.075	0.011	-0.129	0.163	0.95	-0.129	0.162	0.93
	Wave 3	γ_{03}^*	-1	-1.01	-0.01	0.01	0.179	0.178	0.064	-1.367	-0.67	0.97	-1.356	-0.666	0.97
		γ_{13}^*	0	0.006	0.006	0.006	0.186	0.177	0.066	-0.336	0.36	0.96	-0.339	0.351	0.95
		γ_{x3}	0	-0.002	-0.002	-0.002	0.073	0.074	0.011	-0.148	0.144	0.97	-0.147	0.144	0.96
	Wave 4	γ_{04}^*	-1	-1.006	-0.006	0.006	0.198	0.178	0.071	-1.362	-0.664	0.9	-1.357	-0.664	0.91
		γ_{14}^*	0	-0.005	-0.005	-0.005	0.201	0.178	0.072	-0.347	0.35	0.91	-0.35	0.341	0.91
		γ_{x4}	0	-0.014	-0.014	-0.014	0.082	0.075	0.013	-0.161	0.132	0.9	-0.16	0.132	0.93
	Wave 5	γ_{05}^*	-1	-1.022	-0.022	0.022	0.17	0.178	0.061	-1.378	-0.682	0.97	-1.369	-0.681	0.97
		γ_{15}^*	0	0.021	0.021	0.021	0.179	0.177	0.064	-0.318	0.373	0.96	-0.321	0.365	0.96
		γ_{x5}	0	-0.008	-0.008	-0.008	0.076	0.074	0.011	-0.154	0.138	0.95	-0.153	0.137	0.96

Table 6
Summary and comparison of simulation results

		Equal Classes		Unequal Classes	
		MNAR	MCAR	MNAR	MCAR
Sample Size	1500	$ \text{Bias.rel} ^a$	0.022	0.009	0.026
		$ \text{SD.diff} ^b$	0.011	0.008	0.011
		HPD.cvr ^c	0.956	0.941	0.949
1000		$ \text{Bias.rel} $	0.023	0.012	0.032
		$ \text{SD.diff} $	0.012	0.010	0.012
		HPD.cvr	0.950	0.951	0.952
500		$ \text{Bias.rel} $	0.030	0.012	0.068
		$ \text{SD.diff} $	0.015	0.020	0.042
		HPD.cvr	0.952	0.945	0.954

^a The average absolute relative bias across all model parameters, defined by $|\text{Bias.rel}| = \sum_{j=1}^p |\text{Bias.rel}_j|/p$.

^b The average absolute difference between the empirical SDs and the average Bayesian SDs across all model parameters, defined by $|\text{SD.diff}| = \sum_{j=1}^p |\text{SD.emp}_j - \text{SD.avg}_j|/p$.

^c The average coverage probability across all model parameters, defined by $\text{HPD.cvr} = \sum_{j=1}^p \text{HPD.cvr}_j/p$.