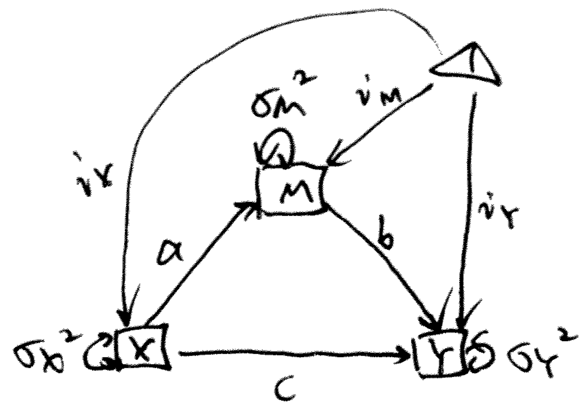
RAM path for SEM model specification.

1. For a path model without latent variables

A, S, F, M matrix



* • A is an asymmetric matrix
  • Include regression pathes.
  • on the row, outcome
  • On the column, input
  • path with one arrow

$$A = \begin{array}{c|c} & \begin{array}{ccc} X & M & Y \end{array} \\ \begin{array}{c} X \\ M \\ Y \end{array} & \begin{pmatrix} 0 & 0 & 0 \\ a & 0 & 0 \\ c & b & 0 \end{pmatrix} \end{array}$$

* • M is a matrix of means and intercepts

$$M = \begin{array}{cc} \begin{array}{c} X \\ M \\ Y \end{array} & \begin{pmatrix} i_X \\ i_M \\ i_Y \end{pmatrix} \end{array}$$

* • S is an symmetric matrix
  • variance or covariances
  • path with two arrows

$$S = \begin{array}{c|c} & \begin{array}{ccc} X & M & Y \end{array} \\ \begin{array}{c} X \\ M \\ Y \end{array} & \begin{pmatrix} \sigma_X^2 & 0 & 0 \\ 0 & \sigma_M^2 & 0 \\ 0 & 0 & \sigma_Y^2 \end{pmatrix} \end{array}$$

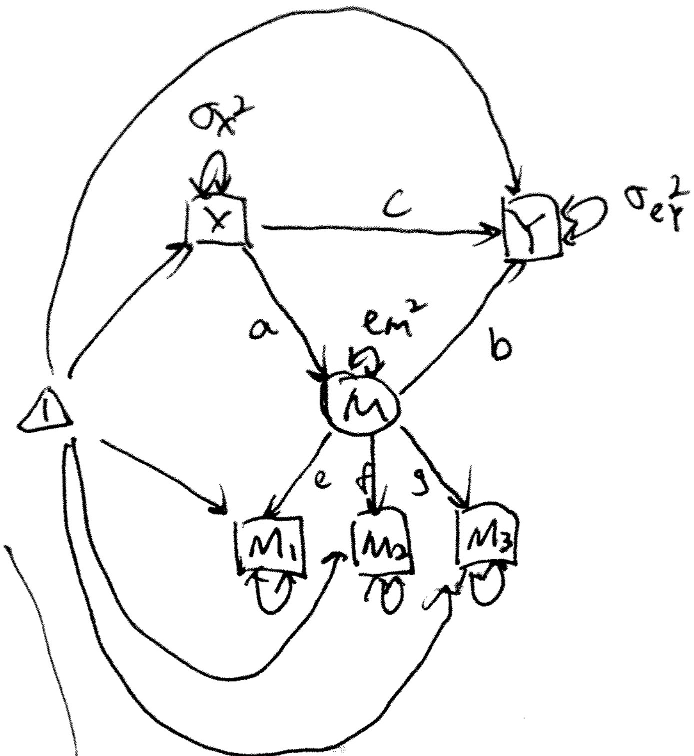
* • F is a filter matrix to identify latent variables.

$$F = \begin{array}{c|c} & \begin{array}{ccc} X & M & Y \end{array} \\ \begin{array}{c} X \\ M \\ Y \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

• on the rows, observed variable only.
• on the columns, both observed and latent
• observed 1, otherwise, 0

2. For a path model with latent variables

$$A = \begin{array}{c|cccccc} & X & Y & M_1 & M_2 & M_3 & M \\ \hline X & 0 & 0 & 0 & 0 & 0 & 0 \\ Y & c & 0 & 0 & 0 & 0 & b \\ M_1 & 0 & 0 & 0 & 0 & 0 & e \\ M_2 & 0 & 0 & 0 & 0 & 0 & f \\ M_3 & 0 & 0 & 0 & 0 & 0 & g \\ M & a & 0 & 0 & 0 & 0 & 0 \end{array}$$



$$S = \begin{array}{c|cccccc} & X & Y & M_1 & M_2 & M_3 & M \\ \hline X & \sigma_X^2 & 0 & 0 & 0 & 0 & 0 \\ Y & 0 & \sigma_{eY}^2 & 0 & 0 & 0 & 0 \\ M_1 & 0 & 0 & \sigma_{eM_1}^2 & 0 & 0 & 0 \\ M_2 & 0 & 0 & 0 & \sigma_{eM_2}^2 & 0 & 0 \\ M_3 & 0 & 0 & 0 & 0 & \sigma_{eM_3}^2 & 0 \\ M & 0 & 0 & 0 & 0 & 0 & \sigma_{eM}^2 \end{array}$$

$$F = \begin{array}{c|cccccc} & X & Y & M_1 & M_2 & M_3 & M \\ \hline X & 1 & 0 & 0 & 0 & 0 & 0 \\ Y & 0 & 1 & 0 & 0 & 0 & 0 \\ M_1 & 0 & 0 & 1 & 0 & 0 & 0 \\ M_2 & & & & 1 & 0 & 0 \\ M_3 & 0 & 0 & 0 & & 1 & 0 \\ \cancel{M} & 0 & 0 & 0 & 0 & & \end{array}$$

$$M = \begin{array}{c|c} X & \mu_X \\ Y & i_Y \\ M_1 & i_{M_1} \\ M_2 & i_{M_2} \\ M_3 & i_{M_3} \\ M & 0 \end{array}$$

With A, S, F, M, the model implied covariance matrix and mean vector can be calculated as

$$\underset{p\times p}{\Sigma} = \underset{p\times(p+q)}{F} * \underset{(p+q)\times(p+q)}{(I-A)^{-1}} * \underset{(p+q)\times(p+q)}{S} * \underset{(p+q)\times(p+q)}{[(I-A)^{-1}]'} * \underset{(p+q)\times p}{F'}$$

$$\underset{p\times 1}{\mu} = \underset{p\times(p+q)}{F} * \underset{(p+q)\cdot(p+q)}{(I-A)^{-1}} * \underset{(p+q)\times 1}{M}$$

Assume we have S and $\bar{Y}$ from data, to get the parameters, we will minimize the function

$$F = \log|\Sigma| + tr(S\Sigma^{-1}) - \log|S| - p \qquad \text{no mean}$$

$$\Rightarrow F = \log|\Sigma| + tr(S\Sigma^{-1}) - \log|S| - p + (\bar{Y}-\mu)'\Sigma^{-1}(\bar{Y}-\mu) \qquad \text{with mean}$$

The derivatives:

$$\frac{\partial F}{\partial \mu(\theta)} = (\bar{Y}-\mu)\Sigma^{-1} = 0 \quad \Rightarrow \mu(\theta) = \bar{Y}$$

gradient

1st. $$\frac{\partial F}{\partial \Sigma(\theta)} = tr\left(\Sigma^{-1}\frac{\partial \Sigma}{\partial \theta}\right) - tr\left(S \cdot \Sigma^{-1}\frac{\partial \Sigma}{\partial \theta}\cdot\Sigma^{-1}\right) = 0$$

2nd $$\frac{\partial^2 F}{\partial \Sigma(\theta)\partial \Sigma(\theta)} = tr\left(\Sigma^{-1}\frac{\partial^2 \Sigma}{\partial \theta \partial \theta}\right) - tr\left(\Sigma^{-1}\frac{\partial \Sigma}{\partial \theta_i}\Sigma^{-1}\cdot\frac{\partial \Sigma}{\partial \theta_j}\right)$$
$$+ tr\left[S\cdot\left(-\Sigma^{-1}\frac{\partial^2 \Sigma}{\partial \theta \partial \theta}\cdot\Sigma^{-1} + \Sigma^{-1}\frac{\partial \Sigma}{\partial \theta_i}\cdot\Sigma^{-1}\frac{\partial \Sigma}{\partial \theta_j}\cdot\Sigma^{-1} + \Sigma^{-1}\frac{\partial \Sigma}{\partial \theta_j}\cdot\Sigma^{-1}\frac{\partial \Sigma}{\partial \theta_i}\cdot\Sigma^{-1}\right)\right]$$