PSY30100-03 -- Assignment 8

Chapter 7:
Inference for Distributions

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Problem 1: 7.16 (p.441)

□ Distribution of the t statistic.

Ans: (See the picture on blackboard). This t distribution has degrees of freedom df=n-1=19. From Table D, we know that 2.5% critical value is 2.093. Thus we reject H0 when t>=2.093 or t=<-2.093.

Upper tail probability p

df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level C

Problem 2: 7.18 (p.441)

☐ One-sided vs. two-sided P-values.

Ans: Because the value of $\bar{x}$ is positive, which supports the direction of the alternative hypothesis $(\mu > 0)$, the P-value for the one-sided test is half as big as that for the two-sided test: p=0.02.

Problem 2: 7.18 (p.441)

- One-sided vs. two-sided P-values.

Additional Question: If the alternative hypothesis becomes $\mu < 0$, then what is the P-value for this one-sided test?

Ans: 98%.

Always sketch the sampling distribution first.

Problem 3: 7.22 (p.442)

□ A final one-sample t test.

$$H_0 : \mu = 20$$

$$H_a : \mu < 20$$

n=115
t=-1.55

(a) df=?
Ans: df=n-1=114

Problem 3: 7.22 (p.442)

- A final one-sample t test.

$$H_0 : \mu = 20$$

$$H_a : \mu < 20$$

n=115

t=-1.55

(b) Between what 2 values does the P-value of the test fall?

					Upper tail probability p							
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
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27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
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z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level C

Problem 3: 7.22 (p.442)

- A final one-sample t test.

(b) Between what 2 values does the P-value of the test fall?

Ans: Using Table D, we refer to df=100. Because $1.290 < |t| < 1.660$, the P-value is between $0.05 < P < 0.10$.

Problem 3: 7.22 (p.442)

☐ A final one-sample t test.

(c) Find the exact P-value.

Ans: R: pt(value, df)

   pt(-1.55, 114)=0.06195664.

Tips: R commands

	normal	t	others
p:probability	pnorm(quantile)	pt(quantile,df)	unif, chisq, lnorm,...
q:quantile	qnorm(prob.)	qt(prob., df)	
d:density	dnorm(quantile)	dt(quantile,df)	
r:random number	rnorm(n)	rt(n,df)	

Problem 4: 7.30 (p.443)

☐ Perceived organizational skills.

(a) Are these data normally distributed?

Ans: The distribution cannot be normal because all values have(presumably) integers between 0 to 4.

Problem 4: 7.30 (p.443)

☐ Perceived organizational skills.

(b) Confidence interval.

Ans: The sample size is quite large (n=282). It should be appropriate to use the 't' method to compute a 99% confidence interval, because the sampling distribution of the sample mean should be approximately t with a large enough sample size (n>40) even if the population distribution is not normal (e.g. very skewed)

Problem 4: 7.30 (p.443)

- ☐ Perceived organizational skills.

(c) Confidence interval.

The one-sample *t*-confidence interval

Steps:

Step1: Confidence level *C* is the area between −*t** and *t**.

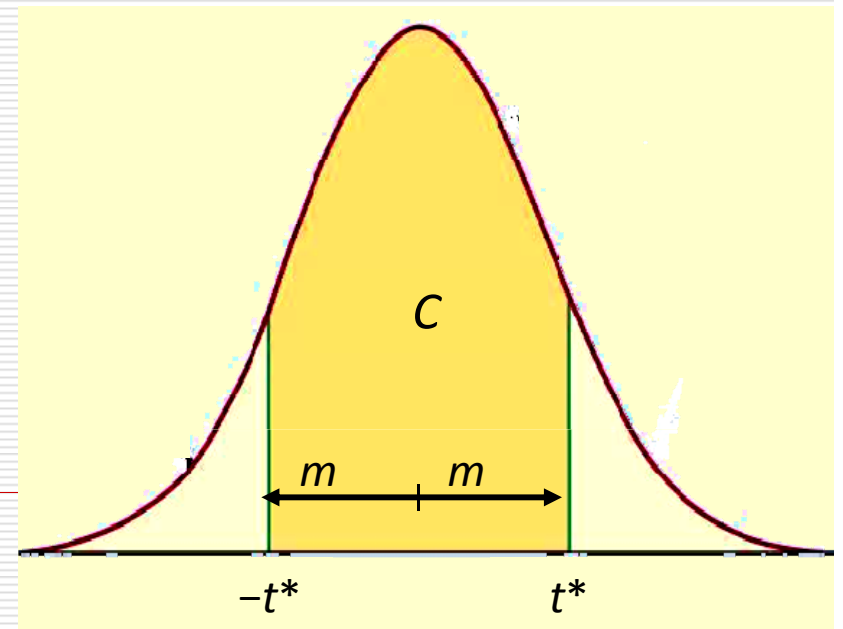*Step 2:* We find *t** in the line of Table D for df = n−1 and *C*.

Step 3: calculate the margin of error *m*

$$m = t^* \times s / \sqrt{n}$$

Step4: Confidence intervals:

$$[\text{estimate - m, estimate + m}]$$

(c) Ans: Steps

Step1: Confidence level *99%* is the area between $-t^*$ and t^*.

Step 2: We find $t^*=2.626$ in the Table D for df = 100 and C=*99%*.

					Upper tail probability p							
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
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28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
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80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
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1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level C

(c) Ans: Steps

Step1: Confidence level *99%* is the area between −*t** and *t**.

Step 2: We find *t**=2.626 in the Table D for df = 100 and C=*99%*.

Step 3: calculate the margin of error *m*

$$m = t^* \times s \big/ \sqrt{n}$$

$$= 2.626 \times 1.03 \big/ \sqrt{282}$$

$$= 0.1610673$$

Step4: Confidence intervals:

$$[\text{estimate - m, estimate + m}]$$

$$-[2.22\text{-}0.1611, 2.22\text{+}0.1611]$$

$$=[2.0589, 2.3811]$$

Problem 4: 7.30 (p.443)

(c) Ans:

	df	t^*	m	interval
Table D	100	2.626	0.1611	[2.0589, 2.3811]
Software qt(p,df)	281	qt(0.995,281) =2.593438	0.1591	[2.0609, 2.3791]

Problem 4: 7.30 (p.443)

☐ Perceived organizational skills.

(d) Generalization.

Ans: The sample might not represent children from other locations well (or, perhaps more accurately, it might not represent well the opinions of the parents of children from other locations.)

Problem 5: 7.80 (p.470)

(use the un-pooled t test by assuming the population variances are not equal)

☐   Independent t test.

source	n	$\bar{x}$	s
Wall Street Journal	66	4.77	1.50
National Enquirer	61	2.43	1.64

(a) Compare two sources of ads.

Steps of a test of significance

Step1: Specify the research question

Step2: Specify the null and alternative hypotheses and a significance level α. Decide on a one-sided or two-sided test

Step3: Calculate the value of the test statistic (pay attention to SE)

Step4: Obtain the p value for the observed data (pay attention to df)

Step5: Interpret the testing result with a significance level α.

Steps of a test of significance

Step1: Specify the research question:

compare the two sources of ads.

Step2: Specify the null and alternative hypotheses and a significance level α. Decide on a one-sided or two-sided test

$$Two-sided : H_0 : \mu_1 = \mu_2 \; vs. \, H_a : \mu_1 \neq \mu_2$$

Step3: Calculate the value of the test statistic

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{4.77 - 2.43}{\sqrt{\dfrac{1.50^2}{66} + \dfrac{1.64^2}{61}}} = 8.368748$$

Steps of a test of significance

Step4: Obtain the p value for the observed data (pay attention to df)

$$df = smallest\ (n_1 - 1,\ n_2 - 1) = 60$$

$$df = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{1}{n_1 - 1}\left(\dfrac{s_1^2}{n_1}\right)^2 + \dfrac{1}{n_2 - 1}\left(\dfrac{s_2^2}{n_2}\right)^2} = \frac{\left(\dfrac{1.50^2}{66} + \dfrac{1.64^2}{61}\right)^2}{\dfrac{1}{66 - 1}\left(\dfrac{1.50^2}{66}\right)^2 + \dfrac{1}{61 - 1}\left(\dfrac{1.64^2}{61}\right)^2}$$

$$= 121.5668$$

Steps of a test of significance

Step4: Obtain the p value for the observed data (pay attention to df)

The P-value is very small (almost 0).

Step5: Interpret the testing result with a significance level α.

The conclusion is: these two sources of ads are significantly different (two-sided test).

Problem 5: 7.80 (p.443)

(use the unpooled t test by assuming the population variances are not equal)

☐ Independent t test.

(b) 95% Confidence Interval of difference.

$$m = t^* \times SE_D = t^* \times \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

df	t^*	Confidence interval
60	2.00	[1.7865, 2.8935]
121.5	1.9797	[1.7808, 2.8992]

Problem 5: 7.80 (p.443)

(use the unpooled t test by assuming the population variances are not equal)

☐ Independent t test.

(c) Conclusion.

Ans: (You may have your own answers)

Advertising in WSJ is seen as more reliable than advertising in the National Enquirer, a conclusion that probably comes as a surprise to no one.