PSY30100-03 -- Assignment 8

Chapter 12:
One-Way Analysis of Variance
(One-Way ANOVA)

TA: Laura Lu
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Definition

Please describe rationale of one-way ANOVA and the F ratio.

- Ans: (You may have your own answers)

One-way ANOVA is the technique used to determine whether more than two population means are equal when there is only one factor or grouping variable in the experiment.

The F-ratio is a ratio which measures the between-group variation compared with the within-group variation. When all group populations have the same standard deviation and the same mean, then this F ratio has the F(DFB, DFW) distribution. When some of group population means are not the same, the F ratio tends to be large.

In a test, if F ratio is large enough, we reject the hypothesis that all group population means are equal.

Problem 1

- A storeowner wishes to compare the average amount of money high school and college students spend on CDs. He randomly selects **ten** students from **three** different student populations: high school students, undergraduate students, and graduate students. The statistical assumptions required to perform a one-way ANOVA to compare the means of these three groups are reasonable based on the data. A partially completed ANOVA table is provided below:

Source	Sum of Squares	DF	Mean Square	*F*
Between				
Within	3240			
Total	4450			

Key points of One-Way ANOVA

- $SS_T = SS_B + SS_W$
- $d.f._B$= #of groups -1

  $d.f._w$= total sample size- #of groups
- $MS_B = SS_B / d.f._B$

  $MS_w = SS_w / d.f._w$
- $F = MS_B/MS_W$

Source	Sum of Squares	DF	Mean Square	*F*
Between	SS_B	K-1	SS_B / d.f.$_B$	MS_B/MS_W
Within	SS_W	N-K	SS_w / d.f.$_w$	
Total	$SS_B + SS_W$			

N: total sample size

K: total number of groups

Problem 1

Source	Sum of Squares	DF	Mean Square	*F*
Between	1210	2	605	5.04
Within	3240	27	120	
Total	4450			

(a) df=?

Ans: B

(b) F=?

Ans: C

(c) Reject null?

Ans: No, because $F_{cv}(0.01,2,27)=5.49>5.04$

Table E *F* distribution critical values

Degrees of freedom in the denominator	*p*	Degrees of freedom in the numerator: 1	2	3	4	5	6	7	8
1	0.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44
	0.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88
	0.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66
	0.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859	5928.4	5981.1
	0.001	405284	500000	540379	562500	576405	585937	592873	598144
2	0.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37
	0.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37
	0.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37
	0.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37
	0.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37
3	0.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25
	0.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85
	0.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54
	0.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49
	0.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62
4	0.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95
	0.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04
	0.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98
	0.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80
	0.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00
5	0.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34
	0.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82
	0.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76
	0.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29
	0.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65
6	0.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98
	0.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15
	0.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60
	0.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10
	0.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03

Problem 2

- I am very interested in the use of technology in the classroom. Suppose we do an experiment in which we teach each of **three** sections of an introductory psychology class in a different way. One section of **six** students receives the standard lecture format (blackboard & discussion), a second section of **six** students receives the same lectures with the addition of overhead transparencies, and the third section receives the lectures on the web. The dependent variable is performance on a standardized (final) exam. Do an analysis by hand to test whether there are different effects of different technology use on test performance. If the F is significant, you should conduct post hoc comparisons (Bonferroni approach).

Subject	GROUP		
	Lecture	Overheads	Web
1	75	85	92
2	72	74	79
3	64	64	78
4	85	85	96
5	59	65	85
6	78	81	80

Review: One-way ANOVA Steps

1. Compute group means and grand mean
2. Compute sums of squared deviations: SS_W and SS_B (maybe SS_T)

(3. maybe: Check to see if $SS_T = SS_B + SS_W$)

4. Compute MS_B (using df_B)
5. Compute MS_W (using df_W)
6. Compute F and p-value (and create ANOVA source table)
7. Compare p to α (or $F_{observed}$ to $F_{critical}$) and decide…

Problem 2

Step 1. Compute group means and grand mean.

Subject	GROUP		
	Lecture	Overheads	Web
1	75	85	92
2	72	74	79
3	64	64	78
4	85	85	96
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6	78	81	80

$M_1=72.167$ $M_2=75.667$ $M_3=85$ GM=77.6

Problem 2

Step 2. Compute SST , SSW and SSB.

We have: M_1=72.167, M_2=75.667, M_3=85, GM=77.6

Using the formulas,

$$SS_B = \sum_{g=1}^{k} n_g (M_g - GM)^2$$

$$= 6[(72.167 - 77.6)^2 + (75.667 - 77.6)^2 + (85 - 77.6)^2] = 528.1$$

$$SS_W = \sum_{g=1}^{k} \sum_{i=1}^{n_g} (x_{ig} - M_g)^2$$

$$= [(75 - 72.167)^2 + (72 - 72.167)^2 + ...] + [(85 - 75.667)^2 + ...] + [...] = 1182.2$$

(Step3: compute SST=1710.3, and Check SST=SSB+SSW)

Problem 2

Step 4. Compute MS_B

$$MS_B = SS_B / d.f._B$$
$$= 528.1/2$$
$$= 264.05$$

Step 5. Compute MS_W

$$MS_w = SS_w / d.f._w$$
$$= 1182.2/15$$
$$= 78.8$$

Problem 2

Step 6. Compute F and p-value

$$F_{obs} = MS_B / MS_w$$
$$= 264.05 / 78.8$$
$$= 3.35$$

p-value:

$0.05 < \underline{P(F>3.35)=0.0627} < 0.1$

Problem 2

Step 7. Compare p to α (or F_{obs} to F_{cv}) and decide…

Compare p to α

$0.05 < \underline{P(F>3.35)=0.0627} < 0.1$

Or use critical value:

$F_{cv}(0.05, 2, 15)=3.68 > 3.35$

$F_{cv}(0.10, 2, 15)=2.70 < 3.35$

Conclusion:

The F is not significant at the level of 0.05.

The F is significant at the level of 0.10.

Bonferroni correction:

- If the F is not significant, we don’t need to conduct any post hoc comparisons.
- But if the F is significant, we should conduct post hoc comparisons. For Bonferroni approach, we need to

(1) conduct 3 independent t tests,

(2) compare each of these three p-values with $\alpha/3$, or compare (3*p-values) with α,

(3) decide which 2 groups are significant different.