PSY30100-03 -- Assignment 5

Chapter 6:

Introduction to Inference

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Question 1.

- Determine whether each of the following statements is true or false.

A) The margin of error for a 95% confidence interval for the mean μ increases as the sample size increases.

B) The margin of error for a confidence interval for the mean μ, based on a specified sample size n, increases as the confidence level decreases.

C) The margin of error for a 95% confidence interval for the mean μ decreases as the population standard deviation decreases.

D) The sample size required to obtain a confidence interval of specified margin of error m increases as the confidence level increases.

Review: margin of error

Confidence interval:

$$\text{point estimate} \pm \text{margin of error}$$

The margin of error shows how accurate we believe our guess is, based on the sampling distribution of the statistic.

Review: confidence level and margin of error

- A confidence interval:

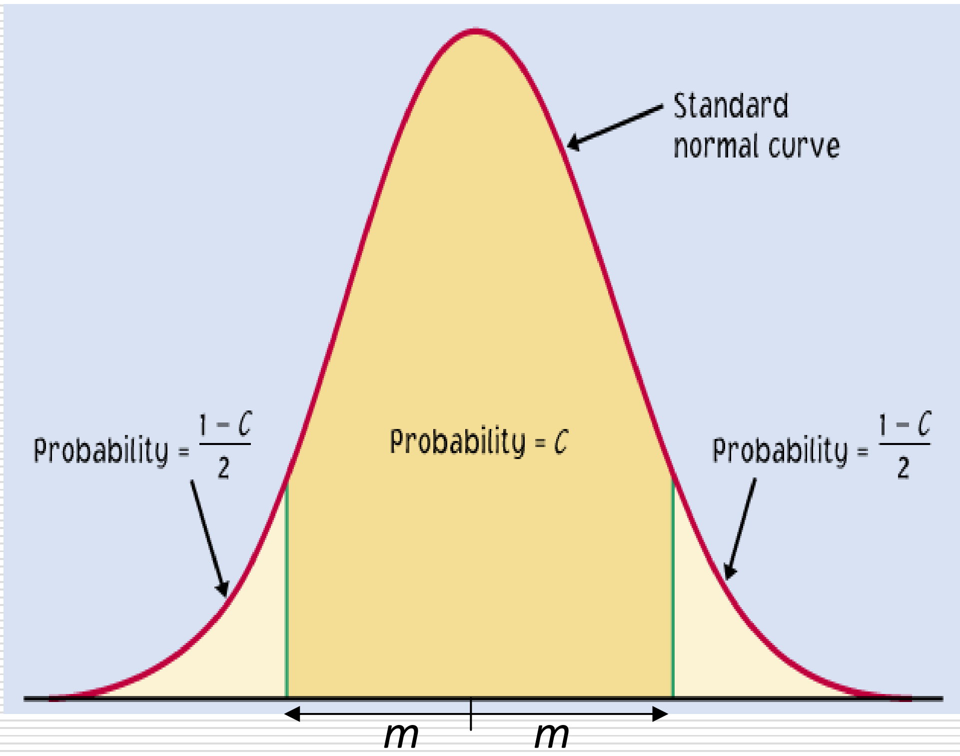
  $$[\overline{x} - m, \quad \overline{x} + m],$$

  where *m* is the **margin of error**

- Two endpoints of an interval

  μ within ($\overline{X} - m$) to ($\overline{X} + m$)

- $$m = z^* \times \frac{\sigma}{\sqrt{n}}$$



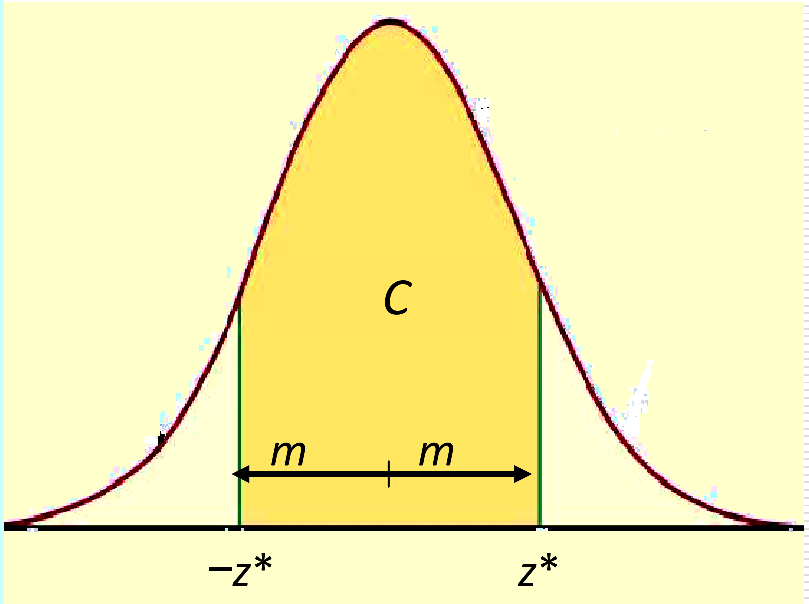
Review: margin of error $m = z^* \times \sigma / \sqrt{n}$

The confidence level *C* determines the value of z^*.

z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											

Higher confidence ***C*** implies a larger margin of error ***m*** (thus less precision in our estimates).

A lower confidence level ***C*** produces a smaller margin of error ***m*** (thus better precision in our estimates).



Get back to question 1.

A) The margin of error for a 95% confidence interval for the mean μ increases as the sample size increases.

Ans: False, because

$$m \downarrow = z^* \times \frac{\sigma}{\sqrt{n \uparrow}}$$

Question 1.

B) The margin of error for a confidence interval for the mean μ, based on a specified sample size n, increases as the confidence level decreases.

Ans: False, because

$$m \downarrow = z^{*} \downarrow \times \frac{\sigma}{\sqrt{n}}$$

Question 1.

C) The margin of error for a 95% confidence interval for the mean μ decreases as the population standard deviation decreases.

Ans: True, because

$$m\downarrow = z^{*} \times \frac{\sigma\downarrow}{\sqrt{n}}$$

Question 1.

D) The sample size required to obtain a confidence interval of specified margin of error *m* increases as the confidence level increases.

Ans: True, because

$$m = z^* \times \frac{\sigma}{\sqrt{n}} \quad \Leftrightarrow \quad n \uparrow = \left(\frac{z^* \uparrow \times \sigma}{m} \right)^2$$

Question 2.

- A nationally distributed college newspaper conducts a survey among students nationwide every year. This year, responses from a simple random sample of 204 college students to the question “About how many CDs do you own?” resulted in a sample mean = 72.8. Based on data from previous years, the editors of the newspaper will assume that $\sigma = 7.2$.

Q: Use the information given to obtain a 95% confidence interval for the mean number of CDs owned by all college students.

Review: Confidence Interval

Confidence interval for a population mean with a given population standard deviation σ.

$$\left[\bar{x} - z^{*} \times \frac{\sigma}{\sqrt{n}},\ \bar{x} + z^{*} \times \frac{\sigma}{\sqrt{n}}\right]$$

Get back to question 2.

- Ans:

  We have n=204, σ = 7.2, $\overline{x}$ = 72.8, and also we can get the z^* score for 95% is $z^* = 1.96$, so the 95% confidence interval for the mean number of CDs owned by all college students is

$$\left[72.8 - 1.96 \times \frac{7.2}{\sqrt{204}},\ 72.8 + 1.96 \times \frac{7.2}{\sqrt{204}}\right]$$

$$= [71.81,\ 73.79]$$

z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Question 3

Answer each of the following questions with yes, no, or can't tell.

A) Does the sample mean lie in the 95% confidence interval?

Ans: Yes, because

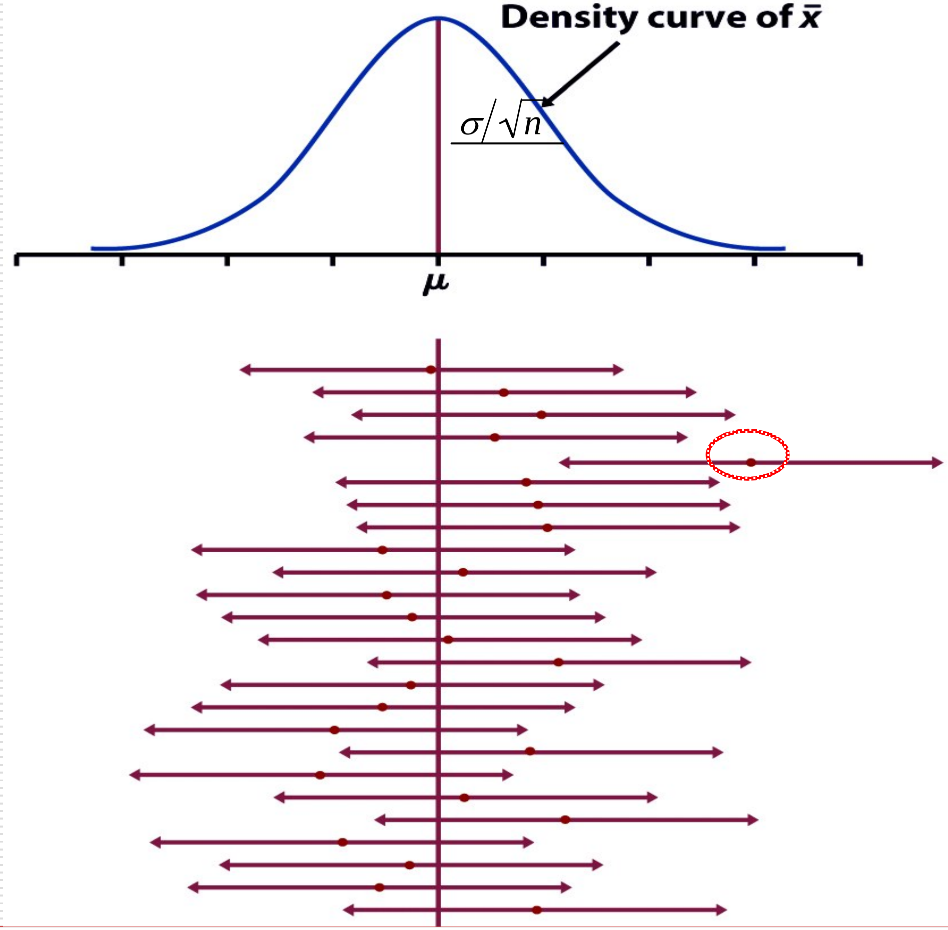
$$(\bar{x} - z^* \times \frac{\sigma}{\sqrt{n}}) \leq \bar{x} \leq (\bar{x} + z^* \times \frac{\sigma}{\sqrt{n}})$$

Question 3.

Answer each of the following questions with yes, no, or can't tell.

B) Does the population mean lie in the 95% confidence interval?

Ans: Can't tell, because the confidence level only shows how confident we are that the procedure will catch the true population parameter, here mean.


Density curve of $\bar{x}$
$\sigma/\sqrt{n}$
μ

Question 3

Answer each of the following questions with yes, no, or can't tell.

C) We were to use a 92% confidence level, would the confidence interval from the same data produce an interval wider than the 95% confidence interval?

Ans: No, because

$$m \downarrow = z^{*} \downarrow \times \frac{\sigma}{\sqrt{n}}$$

Question 3

Answer each of the following questions with yes, no, or can't tell.

D) With a smaller sample size, all other things being the same, would the 95% confidence interval be wider?

Ans: Yes, because

$$m \uparrow = z^{*} \times \frac{\sigma}{\sqrt{n \downarrow}}$$

Question 6.50 (p.390)

- What's wrong?
- (a) $n = 20, \sigma_x = 12, \sigma_{\bar{x}} = \dfrac{12}{20}?$

Ans: The s.d. of the sample mean is

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{12}{\sqrt{20}}$$

Question 6.50 (p.390)

- What's wrong?

- (b) $H_0 : \bar{x} = 10$

Ans: The null hypothesis should be a statement about the population parameter(s), not the sample statistic(s).

Here, the researcher should test μ .

Question 6.50 (p.390)

- What's wrong?

- (c) A study with $\overline{x} = 48$ reports statistical significance for $H_a : \mu > 54$.

Ans: $\overline{x} = 48$ would not make us inclined to believe that $\mu > 54$.

Question 6.50 (p.390)

- What's wrong?
- (d) A researcher tests $H_0 : \mu = 50$ and concludes that the population mean is equal to 50.
- Ans: Even if we fail to reject the H0, we are not sure if H0 is true.

  "fail to reject H0" is different from "know that H0 is true".

  Lack of evidence for rejecting a hypothesis does not imply that we have evidence to support this hypothesis.

Question 6.52b; 6.55 (p.391)

Key words:

- H0: Usually the null hypothesis is a statement of "no effect", "no difference" or "is equal to".

- Ha: The alternative hypothesis is a statement we hope or suspect is true instead of H0, and usually has "higher", "smaller", or "different with".

Question 6.52b; 6.55 (p.391)

- 6.52 (b) The professor believes that the mean μ of the morning class will be higher, so we test

$$H_0: \mu = 72 \quad vs. \quad H_a: \mu > 72$$

- 6.55 (a) key word: higher

$$H_0: \mu = \$62,500 \quad vs. \quad H_a: \mu > \$62,500$$

- 6.55 (b) key word: different

$$H_0: \mu = 2.6\, hours \quad vs. \quad H_a: \mu \neq 2.6\, hours$$

Question 6.56 (p.391)

- Computing the P-value.

Review:

- The P-value is the area under the sampling distribution for values at least as extreme, in the direction of *Ha*, as that of our random sample.

- In order to obtain the P-value, we need
  (1) Z value   (2) Ha (direction of Ha)

- For different Ha, the direction of Ha is different, so the P-value is different.

Review of P-value

- Computing the P-value for

$$H_0 : \mu = \mu_0$$

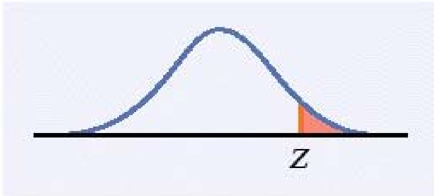
- (a) $H_a : \mu > \mu_0$
- (b) $H_a : \mu < \mu_0$
- (c) $H_a : \mu \neq \mu_0$

Review of P-value (P.383, our textbook)

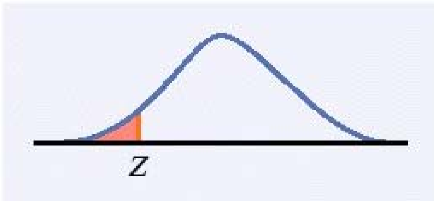
One-sided test

(one-tailed test)

$H_a\colon \mu > \mu_0$ is $P(Z \geq z)$



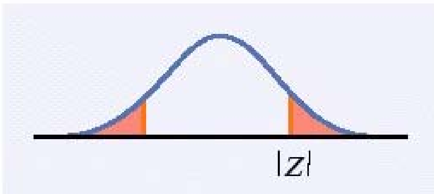
$H_a\colon \mu < \mu_0$ is $P(Z \leq z)$



Two-sided test

(two-tailed test)

$H_a\colon \mu \neq \mu_0$ is $2P(Z \geq |z|)$



To calculate the P-value for a two-sided test, use the symmetry of the normal curve. Find the P-value for a one-sided test and double it.

Question 6.56 (p.391)

- Computing the P-value for

$$H_0 : \mu = \mu_0$$

- (a) $the\ P\ value\ is\ P(Z \geq z = 1.34) = 0.0901$

- (b) $the\ P\ value\ is\ P(Z \leq z = 1.34) = 0.9099$

- (c) $the\ P\ value\ is$

$$2 \times P(Z \geq z = 1.34) = 2 \times 0.0901 = 0.1802$$

Question 6.58 (p.392)

- A two-sided test and the confidence interval.

- The connection (p.388,our textbook):

  "A level α two-sides significance test rejects $H_0 : \mu = \mu_0$ exactly when the value μ_0 falls outside a level $1-\alpha$ confidence interval for μ."

Question 6.58 (p.392)

- A two-sided test and the confidence interval.

- (a) Ans: No.

  30 is not in the 95% confidence interval because $P=0.04 < \alpha=0.05$ means that we would reject H0 at the α level of 0.05.
- (b) Ans: No.

  30 is not in the 90% confidence interval because we would also reject H0 at the α level of 0.10 with $P=0.04$.

Question 6.64 (p.392)

- (Change in California's eighth-grade average science score)

- (You may have your own answers)

  Even if the actual mean score had not changed over time, random fluctuation might cause the mean in 2005 to be different from the mean in 2000. However, in this case the difference was so great that it is unlikely to have occurred by chance; specially, such a difference would arise less than 5% of the time if the actual mean had not changed. We therefore conclude that the mean did change from 2000 to 2005.

Question 6.68 (p.393)

- (Who is the author?)

$$H_0: \mu = 8.9 \quad vs. \quad H_a: \mu > 8.9$$

$$\sigma = 2.5$$

$$\bar{x} = 10.2$$

$$n = 6$$

$$Z = ?$$

$$P = ?$$

$$Conclusion?$$

The Z test

To test the hypothesis $H_0 : \mu = \mu_0$ based on an SRS of size n from a Normal population with unknown mean μ_0 and known standard deviation σ, we rely on the properties of the sampling distribution $N(\mu_0, \sigma/\sqrt{n})$.

1. We first calculate a z-value.

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

2. And then use Table A. The P-value is the area under the sampling distribution for values at least as extreme, in the direction of Ha, as that of our random sample.

Question 6.68 (p.393)

- (Who is the author?)

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{10.2 - 8.9}{2.5 / \sqrt{6}} \approx 1.27$$

$$P = P(Z > 1.27) = 0.1020$$

which means it is not significant at the level of 0.05 or even 0.1, so we can not reject H0.

- Conclusion: There is no enough evidence to reject that these sonnets were written by our poet.