PSY30100-03 -- Assignment 3

Chapter 4: The Study of Randomness

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Question 1

- A card is drawn from an ordinary deck of 52 playing cards. What is the probability that the card is
  1) A club?
  2) A king?
  3) A club and a king?
  4) A club or a king?
  5) Neither a club nor a king?

Review of Probability

- Addition Rule:

  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

  [Special case: $P(A \text{ or } B) = P(A) + P(B)$, when A and B are disjoint]

- Subtraction Rule:

  $P(A) = 1 - P(\text{not } A)$

- Multiplication rule:

  $P(A \text{ and } B) = P(B)P(A|B) = P(A)P(B|A)$

  [Special case: $P(A \text{ and } B) = P(A)P(B)$ when A and B are independent]

Review of Probability

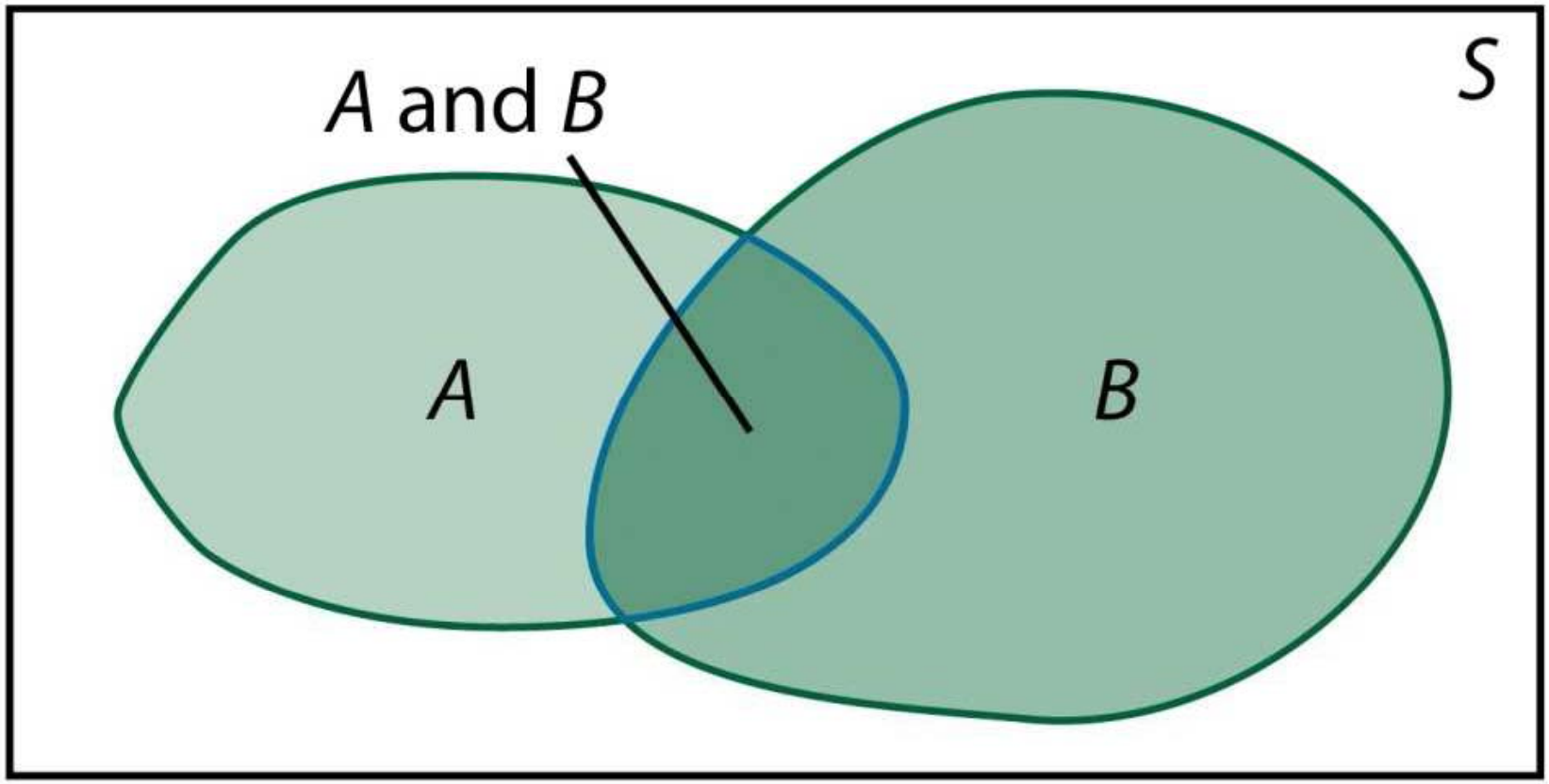
The rule on equally likely outcomes

- If there are N possible equally likely outcomes, then the probability assigned to each is 1/N.
- If an event A consists of N(A) outcomes, then P(A) = N(A)/N

  or

  $$P(A) = \frac{\text{count of outcomes in A}}{\text{count of outcomes in S}}.$$

Question 1



Question 1

- Definitions:
  Event A: the card is a club
  Event B: the card is a king

  1) A club? <=> p(A)?
  2) A king? <=> p(B)?
  3) A club and a king? <=> p(A and B)?
  4) A club or a king? <=> p(A or B)?
  5) Neither a club nor a king?
     => 1- p(A or B)?

Question 1

- Ans:

  1) p(A)?

     N(A)=13

     N=52

     p(A)= N(A)/N=13/52=1/4

Question 1

- Ans:

  2) p(B)?

  N(B)=4

  N=52

  p(B)= N(B)/N=4/52=1/13

Question 1

- Ans:

  3) $p(A \text{ and } B) = p(A) * p(B|A)$

  Because $p(B|A) = 1/13$,

  we have

  $p(A \text{ and } B) = p(A) * p(B|A)$

  $= 1/4 * 1/13$

  $= 1/52$

Question 1

- Ans:

  4) $p(A \text{ or } B) = p(A) + p(B) - p(A \text{ and } B)$

  $= 1/4 + 1/13 - 1/52$

  $= 16/52 = 4/13$

Question 1

- Ans:

  5) 1-p(A or B)

  =1-4/13

  =9/13

Question 2: 4.22

- the tables in 4.21

Blood Type	A	B	AB	O
US Probability	0.40	0.11	0.04	?

- the tables in 4.22

Blood Type	A	B	AB	O
China Probability	0.27	0.26	0.12	0.35

Question 2: 4.22

(1) Event A = the American has type O blood

Event B = the Chinese has type O blood

P(A and B)=?

- Ans:

  Since A and B are independently, we can use the simplified multiplication rule:

  P(A and B)=P(A)*P(B)

  =0.45*0.35

  =0.1575

Question 2: 4.22

(2)Event A = both have type A blood

Event B = both have type B blood

Event C = both have type AB blood

Event D = both have type O blood

- P(A or B or C or D)=?

Question 2: 4.22

- Ans:

  P(A)= 0.40*0.27=0.108

  P(B)= 0.11*0.26=0.0286

  P(C)= 0.04*0.12=0.0048

  P(D)= 0.45*0.35=0.1575

- Since events A, B, C and D are disjoint (mutually exclusive), we can use the simplified addition rule:

  P(A or B or C or D)=P(A)+P(B)+ P(C)+P(D)

  =0.108+0.0286+0.0048+0.1575

  =0.2989

Extension: the general addition rule for more than 2 sets

- **Caution!**

  If events A, B, C and D are not disjoint, then we can't use the simplified addition rule!

- The general addition rule for 3 sets:

  $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$
  $- P(A \text{ and } B) - P(A \text{ and } C) - P(B \text{ and } C)$
  $+ P(A \text{ and } B \text{ and } C)$

- The general addition rule for 4 sets: ...

Question 3: 4.32

- Win: if the winning number contains the digits in your number, in any order.

- (a) There are 6 arrangement of the digits 4, 5, 6 (456, 465, 546, 564, 645, 654),
  so p(win)=6/1000=0.006.

- (b) With digits 2,1,2, there are only 3 distinct arrangements (122, 212, 221),
  so p(win)=3/1000=0.003.

Question 4: 4.64

Review of geometry

- Area (a square)=base*height
- Area (a triangle)=1/2*base*height
- Area (a trapezoid)=1/2*(top base+bottom base)*height

  “The height” must be perpendicular to “the base”!

Question 4: 4.64

Ans:

- (a) There are many ways to verify it.
- (b) p(y<1)=0.5
- (c) p(y<1.5)=1-1/2*1/2*1/2=0.875

  (shaded areas: see blackboard)

Question 5: 4.106

- Known: p(A), p(B), p(A and B)

  To find: p(A or B)?

- The general addition rule:

  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

  $= 0.138 + 0.261 - 0.082$

  $= 0.317$

Question 6: 4.108(based on 4.106)

- There are 4 events,

  1) Draw a Venn diagram;

  2) Indicate each event on the diagram;

  3) Calculate the probability of each event;

  4) Describe in words what each event is.

Question 6: 4.108(based on 4.106)

Ans:

- The Venn diagram: see blackboard
- a) p(A and B)=0.082.
  A household is both prosperous and educated.
- b) p(A^c and B)=p(B)-p(A and B)=0.261-0.082=0.179.
  A household is not prosperous but educated.
- c) p(A and B^c)=p(A)-p(A and B)=0.138-0.082=0.056.
  A household is prosperous but not educated.
- d) p(A^c and B^c)=1-p(A or B)
  =1-(0.082+0.179+0.056)
  =0.683
  A household is neither prosperous nor educated.

Question 7: 4. 110

- Define:

  Event A: an adjusted gross income of at least $100,000

  Event B: an adjusted gross income of at least $1,000,000

  => $A \supset B$

  => p(A and B)=p(B)

- P(B|A)=?

Question 7: 4. 110

Method 1:

- P(A)=(12,757,005)/(312,226,042)
  =0.04085824
- P(B)=(240,128)/(312,226,042)
  =0.0007690838
- P(B|A)=p(A and B)/p(A)
  =p(B)/p(A)
  =0.01882322

Question 7: 4. 110

Method 2:

- Treat event A as a new sample space, then N=12,757,005.

  Since N(B)=240,128

  then p(B)=N(B)/N

  =(240,128)/(12,757,005)

  =0.01882322

Question 8: 4.132 (a)

- About Means & Variances of Discrete Random Variables

Review: Means & Variances of Discrete Random Variables

For a discrete random variable **X** with values x_i, that occur with probabilities $p(x_i)$

- *The mean* of **X** is

$$\mu_X = \sum_{i=1}^{n} x_i \cdot p(x_i)$$

- The *variance of* ***X*** is

$$\sigma_X^2 = \sum_{i=1}^{n} (x_i - \mu_X)^2 \, p(x_i)$$

Question 8: 4.132 (a)

Ans:

- Mean

$$\mu_X = 1\times 0.2 + 2\times 0.6 + 3\times 0.2 = 2$$

- Variance

$$\sigma_X^2 = (1-2)^2\times 0.2 + (2-2)^2\times 0.6 + (3-2)^2\times 0.2$$

$$= 0.4$$

$$\sigma_X = \sqrt{0.4} = 0.6325$$