PSY30100-03 -- Assignment 2

Chapter 1: Describing Distributions with Numbers, Density Curves and Normal Distributions

TA: Laura Lu

September 7, 2009

Question 1

1. Calculate the sample means, sample medians, and sample standard deviations of the following two data sets. Please notice that each value of the second data set is obtained by multiplying the corresponding value of the first data set by -1 and then adding 3. Comment on the relationship between the two means, two medians the two standard deviations.

- Data set 1: 1 3 6 9 10;
- Data set 2: 2 0 -3 -6 -7

Question 1

Data set 1: 1 3 6 9 10;

Using definitions:

a) sample mean

$$= \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{5}(1+3+6+9+10)=5.8$$

b) sample median=6

Question 1

c) sample standard deviation

$$= \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$= \sqrt{\frac{1}{5-1}[(1\text{-}5.8)^2 + (3\text{-}5.8)^2 + (6\text{-}5.8)^2 + (9\text{-}5.8)^2 + (10\text{-}5.8)^2]}$$

$$\approx 3.8$$

Question 1

Data set 2: 2 0 -3 -6 -7

Two ways:

The first way: using definitions

a) sample mean=-2.8
b) sample median=-3
c) sample standard deviation≈3.8

Question 1

The second way: using linear transformation!

Data set 1: 1 3 6 9 10 -> x
Data set 2: 2 0 -3 -6 -7 -> y

each y value is obtained by first multiplying the corresponding x value by -1 and then adding 3

Linear transformation (a=3, b=-1)

$$y = 3 + (-1)x$$

Recap

Properties of linear transformations

mean(new)=a+b*mean(original)

median(new)=a+b*median (original)

sd(new)=|b|*sd(original)

Question 1

Data set 1: 1 3 6 9 10;
Data set 2: 2 0 -3 -6 -7

	sample mean	sample median	sample standard deviation
Data set 1	5.8	6	≈ 3.8
Data set 2	a+b*5.8 =3-1*5.8 =-2.8	a+b*6 =3-1*6 =-3	≈|b|*3.8 =1*3.8 =3.8

Question 2: problem 1.68 (p.50)

- Be careful about how you treat the zeros.

- Keep the whole sample!
- Do not delete the member in the sample which has a value zero.

Ans: (Lot of answers, you can have your own)

0 0 20,000

Omit zeros → median: 20,000; mean: 20,000

Place 0 by 14,000 → new median: 14,000; new mean: 16,000

Question 3: problem 1.111 (p.72)

A strategy to distinguish mean, mode, and median:

Step 1. Symmetric or not.

Step 2. Symmetric: three values take the same point.

Non-symmetric: 3 Steps:

(1) Find the highest peak: the mode.

(2) The median always stays in the middle.

(3) The mean always moves away from the median towards the longer tail.

Question 3: problem 1.111 (p.72)

Ans:

a) Non-symmetric: B: the median. C: the mean.

b) Symmetric: A: the median. A: the mean.

c) Non-symmetric: B: the median. A: the mean.

Question 4: problem 1.121 (p.74)

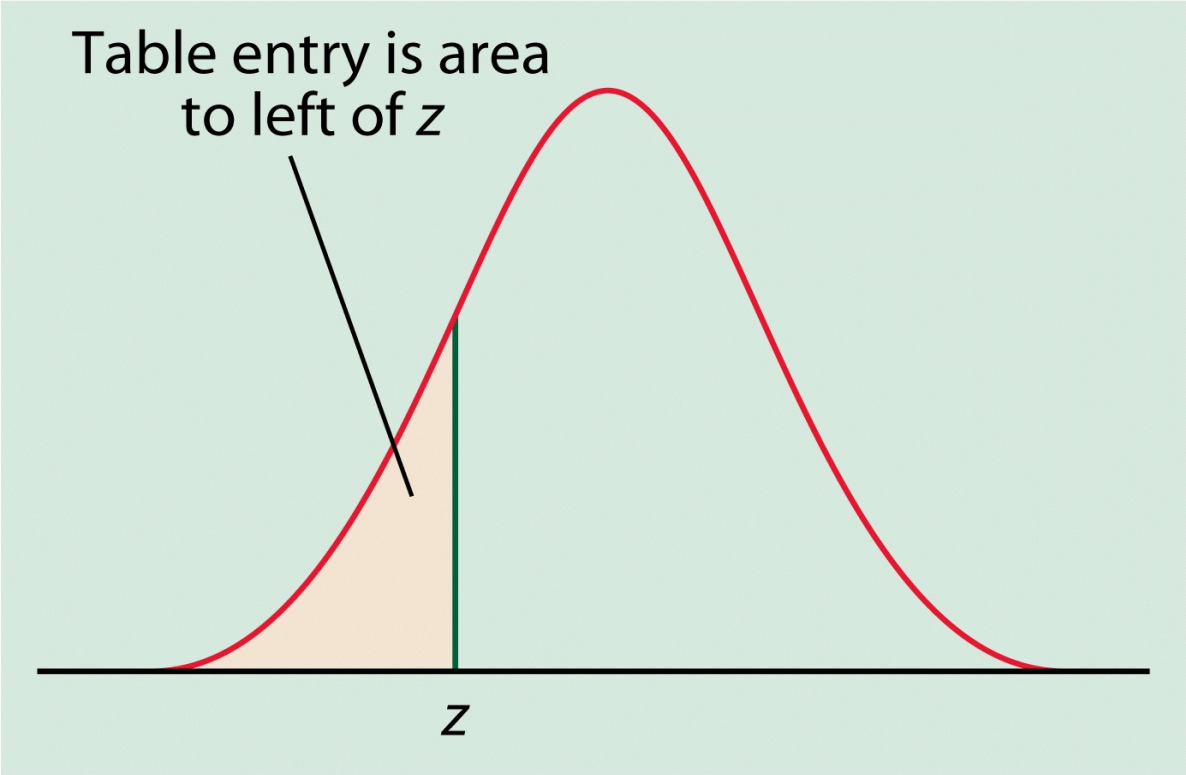
a) $Z \leq -1.9$

b) $Z \geq -1.9$

c) $Z > 1.55$

d) $-1.9 < Z < 1.55$

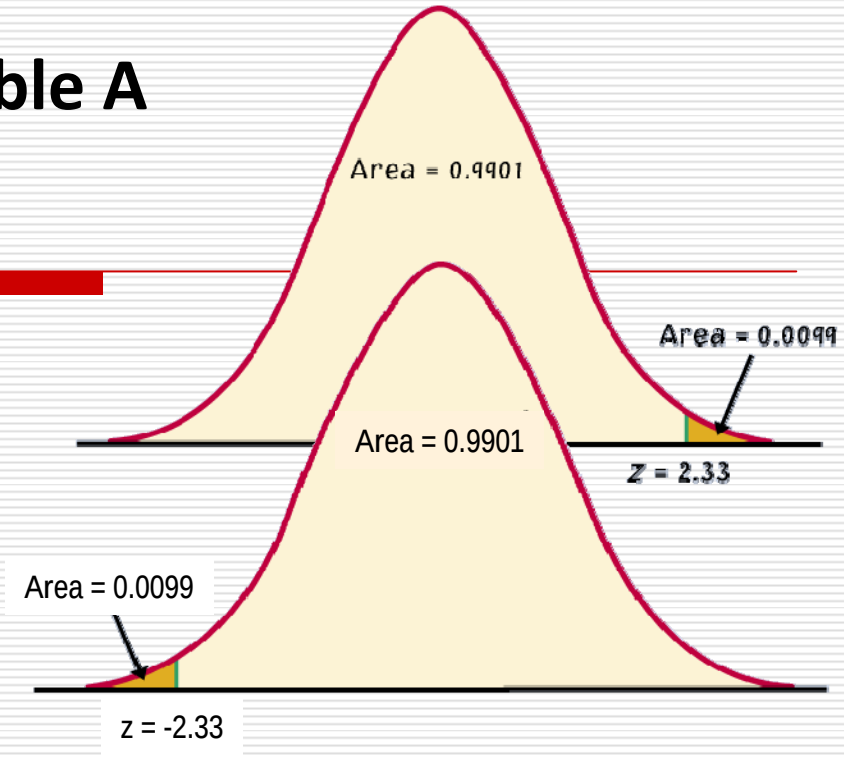
Table A gives the area under the standard Normal curve to the left of any z value: cumulative proportion.



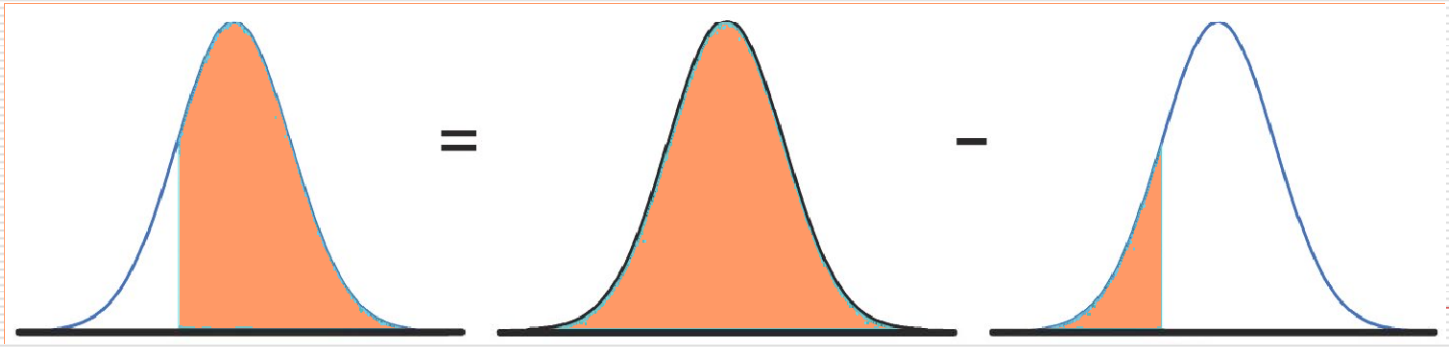
To use table A, we must have a variable with a normal distribution whose mean is 0 and whose standard deviation is 1.

Recap: Tips on using Table A

Because the Normal distribution is symmetrical, there are 2 ways that you can calculate the area under the standard Normal curve <u>to the right</u> of a z value.



area right of z = area left of -z



area right of z = 1 - area left of z

TABLE A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.00	.0045	.0044	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.00	.0060	.0059	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.00	.0080	.0078	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.01	.0104	.0102	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.01	.0136	.0132	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.01	.0174	.0170	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681

.0606 is the area under N(0,1) left of z = -1.55

.0287 is the area under N(0,1) left of z = -1.9

Question 4: problem 1.121 (p.74)

a) $Z \leq -1.9$ ->check table A =.0287

b) $Z \geq -1.9$ ->1-area($Z < -1.9$)=1-.0287=.9713

c) $Z > 1.55$ ->area($Z < -1.55$)=.0606

d) $-1.9 < Z < 1.55$

-> area($Z < 1.55$) - area($Z < -1.9$)

= [1-area($Z < -1.55$)] - area($Z < -1.9$)

= .9394-.0287=.9107

Question 5: problem 1.124 & 1.125 (p.74)

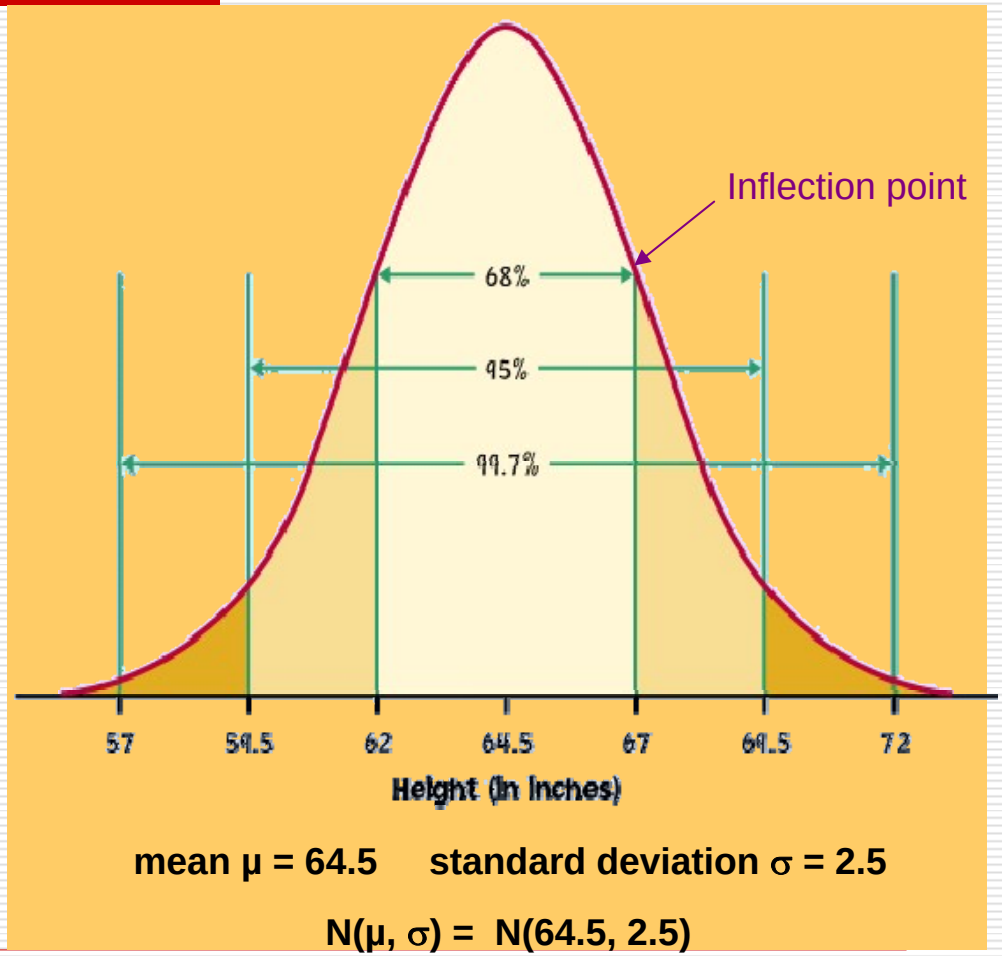
1. Usually, 2 steps
   1) Standardize x to a Z-score using the following formula $$z = \frac{x - \mu}{\sigma}$$
   2) Check Table A for the percentage

2. For some special cases, we can use "The 68-95-99.7 Rule"

Recap: The 68-95-99.7 Rule

All Normal curves N(μ,σ) share the same properties

- About 68% of all observations are within 1 standard deviation (σ) of the mean (μ).
- About 95% of all observations are within 2 σ of the mean μ.
- Almost all (99.7%) observations are within 3 σ of the mean.



mean μ = 64.5 standard deviation σ = 2.5

N(μ, σ) = N(64.5, 2.5)

Question 5: problem 1.124 & 1.125 (p.74)

1.124 Ans:

$$z = \frac{x - \mu}{\sigma} = \frac{70 - 100}{15} = -2$$

using "The 68-95-99.7 Rule", the answer is 2.5%.

1.125 Ans:

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = 2$$

using "The 68-95-99.7 Rule", the answer is 2.5%.

Question 6: problem 1.126 & 1.129 (p.74)

1.126 Ans:

Tonya: $$z = \frac{x-\mu}{\sigma} = \frac{1320-1026}{209} \approx 1.4067$$

Jermaine: $$z = \frac{x-\mu}{\sigma} = \frac{28-20.8}{4.8} \approx 1.5$$

1.129 Ans:

Maria: $$z = \frac{x-\mu}{\sigma} = \frac{29-20.8}{4.8} \approx 1.7083$$

$$z = \frac{x-\mu}{\sigma} \Leftrightarrow x = z\sigma + \mu \approx 1.7083 \times 209 + 1026 = 1383$$